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SCALING FOR SHOCK RESPONSE OF EQUIPMENT IN DIFFERENT SUBMARINES

N00014-91-J-4059

by

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Technical Report No.92-2

December 1992

UNIVERSITY OF MARYLAND AT COLLEGE PARK

DEPARTMENT OF MECHANICAL ENGINEERING

December 9, 1992

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Sincerely,

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ABSTRACT

This report presents scaling rules developed to predict the response of submarine equipment subjected to underwater chemical explosions. The computer was used as a surrogate for shock tests by employing the University of Maryland HULL code. A simplified model of a hull section was used to contain a frame-mounted single-degree of freedom equipment. A general scaling rule has been developed to handle the spread in the shock response attributable to the charge size, equipment weight, and equipment frequency. In this report the shock response is considered to be the absolute maximum acceleration of the equipment mass as a function of the shock factor (square root of the energy flux density) for a given charge weight. The report also examines those cases where a new hull is derived from an original hull by the linear scaling law. The solution of the shock response is well known when the internal equipment has also been linearly scaled. A new general scaling rule is developed for those cases when the equipment is not linearly scaled, i.e., the equipment and charge weight used in the original hull remains unchanged when installed in the linearly-scaled hull or a completely different equipment and charge weight are used with the new hull. It is shown how this new general scaling rule can be used for either linear or parabolic scaling. It is emphasized that the test sections were short and devoid of typical equipment present in a real compartment. The results, nevertheless, provide trends and ratios in shock design values, not necessarily absolute design numbers. The approach taken in developing these scaling rules could be useful for enhancing field data that may exist for a given class of boat, so as to allow greater usage of these data for different equipment subject to a variety of charge weights, attack geometries, and other boats.

BACKGROUND

Previous analysis and studies at the University of Maryland and elsewhere have illustrated the difficulty in relating the equipment response at different charge weights for the same shock factor. An early study [1] showed a promising scaling rule that appears valid over a wide range of charge sizes for the same hull and equipment. Another report [2] examined how far this range might be extended for both lower charge weights and higher charge weights; compared linear and parabolic least square fits of the data which are in the form of equipment peak acceleration response versus shock factor; introduced new scaling rules for equipment weight and equipment frequency for single-degree of freedom equipment; and pointed out the hazards of extrapolating over a wide range of shock factor using a limited range of data.

The current report examines in greater detail the range of application of the these previously developed scaling rules for new and more comprehensive response data so as to either confirm or set new bounds on the charge weights, equipment weights, and equipment frequencies. Linear scaling between different hulls is also examined, where in general, the hull geometry, equipment weight and frequency, and charge size are all scaled by a linear factor. A new scaling rule is also developed for the case where the hull geometry is scaled linearly but the equipment either remains unchanged or a completely different equipment is installed in the new hull.

Model Hulls

Two different model submarine hulls were employed in the study, each designed for approximately the same depth. Models B and F represent a 33-foot diameter hull and a 40.29-foot diameter hull, respectively. The diameters, geometrical layout, and scantlings for each hull are shown in Fig.1. An earlier study [1] demonstrated that a five frame model is adequate for the purpose of this kind of investigation. The University of Maryland "HULL" code, which has been described elsewhere [3], is the principal means used in the creation of the mathematical models. This code calculates the time response of the equipment and their base supports internal to a submarine-like ring-stiffened pressure hull when the hull is subjected to an underwater chemical explosion. The pressure hull, the underwater explosion, and the fluid-structure interaction are all modelled with sufficient detail to provide a realistic environment for the study of shock excited internal equipment. This resulted in models B and F each having more than 1,100 degrees of freedom.

Equipment Response

The absolute acceleration of the equipment mass as a function of the shock factor for a given charge weight is the measure of response and its variation is examined to establish trends that may affect equipment design. Figure 2 is a schematic of the shot geometry where the depth of the center line of the hull and the charge are always held at 60 feet so that the cavitation pressure remains the same in all cases. Neutral buoyancy is always maintained.

The measure of the shock intensity used herein is the square root of the acoustic approximation of the energy flux density, or shock factor SF, where

$$SF=\frac{\sqrt{Q}}{R} \tag{1}$$

as shown by Cole [4]; Q is the charge weight in pounds of TNT, and R is the distance in feet between the hull and the charge.

LINEAR SCALING

Linear scaling is the process whereby a new hull is sized from an existing hull by the linear ratio of the hull diameters.

Thus, the hull thickness and all other scantlings are scaled by the same linear scaling factor. This approach assumes a thin-walled cylinder under uniform pressure, whereby the tangential stress and the longitudinal stress are both proportional to the diameter of the cylinder and inversely proportional to the thickness of the cylinder. Consequently, if d is the diameter of the prototype hull, and the diameter D of the scaled hull, then

$$D=Ld \tag{2}$$

where L is the linear scaling factor. This scaling factor is also used to scale the charge size, standoff distance, and the equipment from the prototype hull to the scaled hull.

The motion of an equipment that is created by some excitation requires the linear scaling factor to be extended to time and the kinematic descriptors of the response. Thus, if t is time for the motion of the equipment in the original hull, then the time T for the new equipment in the scaled hull is

$$T=Lt \tag{3}$$

The relationship between the existing velocity v, acceleration a, and frequency f and their corresponding quantities in the scaled hull, expressed in upper-case notation, are

$$V=v \tag{4}$$

$$A=\frac{a}{L} \tag{5}$$

$$F=\frac{f}{L} \tag{6}$$

The linear scaling relationship between the prototype's chemical charge weight q and shock factor sf and the corresponding scaled quantities are

$$Q=L^3q \tag{7}$$

$$SF=\sqrt{L}sf \tag{8}$$

The general linear scaling law was applied to model B which formed the prototype hull containing a 20-kip, 20-Hz equipment as shown in Fig.1(a). The velocity of the equipment relative to its moving base point was obtained for a 1740-lb charge weight and a shock factor of 0.15. Two linearly scaled models were constructed for a 33-foot diameter boat, called the SBS model, and for a 40-foot diameter boat, called the LBS model. The equipment weight, equipment frequency, charge weight and shock factor for the SBS model were calculated as 9.782 kips, 25.385 Hz, 851 lb and 0.133, respectively, and the corresponding values for the LBS model were 35.618 kips, 16.500 Hz, 3098.8 lb and 0.165. An overlay of the relative velocity response of each equipment in models B, SBS, and LBS, is shown in Fig.3. Note that eq.(3) was used to scale time for the response of the SBS and SBL models. The closeness of fit of these responses supports the validity of the general linear scaling law as well as the adequacy of the performance of the HULL code.

CHARGE WEIGHT SCALING

Consider an equipment attached to a given hull as shown in Fig.1 subject to a fixed charge weight and a varying shock factor. Figure 4 is a plot of two typical linear least square fits through the equipment response. As observed earlier [1], the acoustic pressure appears to be the key variable that affects the equipment response. A scaling rule was constructed by dividing the slope of the line representing the least squares fit of the data for the charge weight Q_a by the acoustic pressure, where the acoustic pressure is defined as $Q_a^{3/8}/R$. Thus,

$$s_a \frac{R}{Q_a^{(3/8)}} = s_a \frac{Q_a^{(1/8)}}{SF} \tag{9}$$

where s_a is the slope of the least square line for charge weight Q_a. For equal shock factor, the slope s_b for charge weight Q_b, is related to the slope for charge weight Q_a as

$$s_b = s_a \left(\frac{Q_a}{Q_b}\right)^{(1/8)} \tag{10}$$

This scaling rule for charge weight was tested in an earlier study [2]. For example, consider Fig.5 which shows the response data plotted for model B with a 20-kip, 20-Hz equipment mounted to the hull frame. Straight lines join the data points for charge weights ranging from 600 lb to 3,625 lb. Figure 6 shows the linear least square fit for each charge weight data in Fig.5. Note that the symbols are used to identify each line more clearly and, consequently, are not data points. Figure 7 shows all of the slopes scaled to the 1,160-lb charge by the scaling rule of eq.(10).

Although an examination was made to alter the exponent in eq.(10) to narrow the range of the scaled slopes even further, the results proved that there was little to be gained by deviating from a rule based on physical reasoning. Consequently, after reviewing all of the data it was concluded that eq.(10) would serve as a very good predictor for charge weights ranging from 600-lb to 3,625-lb of TNT.

The response data for model B, which is the prototype, and model B's scaled counterparts are summarized for this range of charge weights in Appendix A. The notation SB and LB refer to a linearly scaled hull with the originally installed equipment in model B, while the notation SBS and LBS refer to the cases where the general linear scaling law was used as presented in the previous section. L = 26/33 for the SB and SBS models, and L = 40/33 for the LB and LBS models. Similar data are presented for model F in Appendix B, where L = 30.83/40.3 for the SF and SFS models, and L = 52.4/40.3 for the LF and LFS models. These data were utilized in developing the additional scaling rules that follow.

EQUIPMENT WEIGHT SCALING

Having established a scaling rule for the response slopes for different charge weights for a given hull as expressed by eq.(10), the next step in the analysis was to examine how these response slopes might be modified for different equipment weights and equipment fixed base frequencies. Consider the following general scaling rule to find the response slope s_b for equipment weight W_b knowing the slope s_a for equipment weight W_a:

$$s_b = s_a \left(\frac{W_a}{W_b} \right)^n \tag{11}$$

An earlier investigation of this rule [2] reasoned that because two cubes of different weight scale by a factor of 1/3, and the response data are plotted as a function of the square root of the charge weight as shown in eq.(1), then n = 1/6 might prove to be a good estimate for the exponent n. This scaling rule was examined in the current study over a wide range of conditions for models B and F. Consider the data for model B in Table 1. The equipment weight ranged from 15 kips to 35 kips, while the equipment frequency was held to either 20 Hz or 30 Hz. Three charge weights and five shock factors ranging from 0.15 to 0.75 in 0.15 increments were used. The data in the table are the actual peak acceleration response and the slope of the least square fit through the response data for the given charge weight. Table 2 contains the exponent n in eq.(11) that would be required to satisfy the actual slopes obtained from the computer results in Table 1 for all combinations of the weight ratios. In the case of the 20-Hz equipment, an overall average of the data for the three charge weights produced n = 0.1648, and n = 0.2417 for the 30-Hz equipment.

Further examination of the data revealed that eq.(11) was relatively insensitive to the exponent n. For example, eq.(11) was tested against the data in Table 1 by using n = 1/6 and the 25-kip equipment as the reference weight. The results are summarized in Table 3. As an example of the calculations, refer to the data in Table 1 where the response slope equals 54.0409 for the 15-kip, 20-Hz equipment subject to an 1160-lb charge weight. Thus, s_a = 54.0409, W_a = 15 kips, and W_b = 25 kips. Therefore,

$$s_b = 54.0409\left(\frac{15}{25}\right)^{\left(\frac{1}{6}\right)} = 49.6304 \tag{12}$$

This value is found in Table 3 and the percent error is computed between this scaled slope and the actual slope of 49.7273, yielding an error of 0.19%. The largest error for the range of parameters is 4.27%.

Similar results for model F are shown in Tables 4-6 where the highest error shown in Table 6 for n = 1/6 was 2.22%, again suggesting the relative insensitivity of eq.(11) to changes in the exponent n. Consequently, the exponent n = 1/6 was judged satisfactory for equipment ranging from 15 kips to 35 kips, and frequencies in the neighborhood of 20 Hz and 30 Hz.

EQUIPMENT FREQUENCY SCALING

A scaling rule for the equipment fixed base natural frequency was developed in a similar way. Consider the general form of the rule as follows:

$$s_b = s_a\left(\frac{f_b}{f_a}\right)^n \tag{13}$$

where

s_a = reference slope for a given charge weight, equipment weight, and equipment frequency f_a;

s_b = slope for the same charge weight, equipment weight, but the equipment frequency is now f_b.

Table 7 summarizes the response data for model B subject to three charge weights, where the equipment frequency ranges from 15 Hz to 35 Hz in increments of 5-Hz, and the equipment weight ranges from 15 kips to 25 kips in 5-kip increments. The required values of n in eq.(13) that satisfy these data are summarized in Table 8 for all combinations of the frequencies. The overall averages shown in this table for the 15-kip, 20-kip, and the 25-kip equipment weights were 1.6610, 1.6161, and 1.5649, respectively. Table 9 shows the scaled slopes for n = 1.6 using the 25-Hz equipment as the reference frequency. For example, a slope of 32.6784 is found in Table 7 for the 15-kip, 15-Hz equipment. Thus, the predicted slope s_b is

$$s_b = 32.6784\left(\frac{25}{15}\right)^{(1.6)} = 73.9978$$

The error between this scaled value and the actual slope is 4.93%. The results in Table 9 shows errors at less than 5%, except for three instances above this value, the largest being 6.74%.

Tables 10-12 show similar results for model F over the same range in the parameters. The analysis of all combinations of frequencies shown in Table 11 yielded the following overall average values of n equal to 1.6489, 1.6070, and 1.5820, for 15-kip, 20-kip, and 25-kip equipment weights, respectively. A value of n = 1.6 was again used to predict the errors against the 25-Hz equipment as shown in Table 12. The results show errors less than 5% except for the 15-kip, 15-Hz equipment, the largest error for all the data being 6.30%. Consequently, based on these results for models B and F, n = 1.6 was selected for scaling on frequencies ranging from 15 Hz to 35 Hz.

GENERAL SCALING RULES

Same Hull

The scaling rules expressed by eqs.(10), (11), and (12), can be and are combined to form the general scaling rule for the same hull:

$$s_b = s_a\left(\frac{Q_a}{Q_b}\right)^{\left(\frac{1}{6}\right)}\left(\frac{W_a}{W_b}\right)^{\left(\frac{1}{6}\right)}\left(\frac{f_b}{f_a}\right)^{(1.6)} \qquad (14)$$

where n = 1/6 is used for the equipment weight scaling exponent, and n = 1.6 for the frequency scaling. The ranges used for applying this rule are: 600 lb to 3,625 lb for the charge weight; 15 kips to 35 kips for the equipment weight; and 15 Hz to 35 Hz for the equipment fixed base frequency.

General Linear Scaling Law

Consider the hull of diameter d shown in Fig.8(a) which includes an equipment of weight w and frequency f subject to a charge weight q at a distance r from the hull. The slope of the response is

$$s=\frac{a}{(sf)}=\frac{ar}{q^{(1/2)}} \tag{15}$$

where a is the peak acceleration experienced by the equipment. Figure 8(b) represents the linearly scaled model of Fig.8(a), such that the hull, scantlings, equipment, and charge weight are scaled by the factor L = D/d. It follows that

$$S=\frac{A}{(SF)}=\frac{AR}{Q^{(1/2)}} \tag{16}$$

Using eqs.(2), (5), and (7), we obtain the general scaling rule for the response slopes as:

$$S=\frac{s}{L^{(3/2)}} \tag{17}$$

Linearly Scaled Hull - Same Equipment, Same Charge Weight

Figure 9 shows a sequence of three hulls. The original hull in Fig.9(a), subject to a charge weight q, supports an equipment of weight w and frequency f. Assume the response slope s is known. Figure 9(b) shows the original configuration that has been linearly scaled, where the equipment weight and frequency are W and F, respectively, and the charge weight is Q. The response slope is S and is related to s by eq.(17). Figure 9(c) shows the hull geometry as being the same as in Fig.9(b), but the equipment and charge weight are the same as the hull in Fig.9(a). Let S' be the response slope for the model in Fig.9(c). It follows that

$$S'=S\left(\frac{Q}{q}\right)^{\left(\frac{1}{8}\right)}\left(\frac{W}{w}\right)^{\left(\frac{1}{6}\right)}\left(\frac{f}{F}\right)^{(1.6)} \tag{18}$$

$$S'=\frac{s}{L^{(3/2)}}(L^{3})^{(1/8)}(L^{3})^{(1/6)}(L)^{(1.6)} \tag{19}$$

$$S'=sL^{(0.975)} \tag{20}$$

Thus, knowing the response slope s for the equipment in the original model in Fig.9(a), one can predict the response slope S' when this equipment is subject to the same shock when placed in the scaled hull in Fig.9(c).

Linearly Scaled Hull - Different Equipment, Different Charge Weight

Consider the same sequence of hulls in Fig.9. Assume that the response slope s is known for the model in Fig.9(a) and that Fig.9(b) is again the linearly scaled model. Suppose an equipment and/or charge weight are different from those in the prototype in Fig.9(a). Equation (19) can be used to predict the response slope for this new and different model. Thus, one could have response data for a particular boat modelled in Fig.9(a) and predict the response for different charge weight, equipment weight, and equipment frequency, respectively, for the model in Fig.9(c). Thus, one could have response data for a particular boat modelled in Fig.9(a) and predict the response for different equipment in a linear scaled hull.

PARABOLIC SCALING

The Intercept Rule

Consider a typical plot of both the linear fit and the parabolic fit through the same set of data (x_i, y_i) as shown in Fig.10. The least squares fit of a straight line is

$$y=Cx \tag{21}$$

where

$$C=\frac{\left(\Sigma x_i y_i\right)}{\left(\Sigma x_i^2\right)} \tag{22}$$

The parabola is expressed by

$$y=Ax+Bx^2 \tag{23}$$

where

$$A=\frac{\left(\left(\Sigma x_i y_i\right)\left(\Sigma x_i^4\right)-\left(\Sigma x_i^3\right)\left(\Sigma x_i^2 y_i\right)\right)}{D} \tag{24}$$

$$B=\frac{\left(\left(\Sigma x_i^2\right)\left(\Sigma x_i^2 y_i\right)-\left(\Sigma x_i^3\right)\left(\Sigma x_i y_i\right)\right)}{D} \tag{25}$$

$$D=(\Sigma x_i^2)(\Sigma x_i^4)-(\Sigma x_i^3)^2 \tag{26}$$

It is interesting to observe that the point of intersection of the two curves at (x_c, y_c) is such that

$$x_c=\frac{(\Sigma x_i^3)}{(\Sigma x_i^2)} \tag{27}$$

Thus, x_c has the same value for each set of data provided the x_i values are identical.

Same Hull

Suppose there exists a parabolic least squares fit through a set of data as shown in Fig.11. Consider any two points on the curve: (x_1, y_1) and (x_2, y_2). The equation for the parabola that passes through these two points is

$$y=Ex+Fx^2 \tag{28}$$

where

$$E=\frac{(y_1x_2^2-y_2x_1^2)}{(x_1x_2(x_2-x_1))} \tag{29}$$

$$F=\frac{(x_1y_2-x_2y_1)}{(x_1x_2(x_2-x_1))} \tag{30}$$

Note that the x-coordinates represent the shock factor and the y-coordinates are the maximum acceleration responses of the equipment. If $x_1 = 0.3$ and $x_2 = 0.6$, eqs.(29) and (30) reduce to

$$E=\frac{(4y_1-y_2)}{(0.6)} \tag{31}$$

$$F=\frac{(y_2-2y_1)}{(0.18)} \tag{32}$$

Equation (28) may be used to generate a parabola through two new values Y_1 and Y_2, where Y_1 and Y_2 are coordinates obtained by applying eq. (14). For example, suppose we have the least squares parabola through the data for a system composed of charge weight Q_a, equipment weight W_a, and frequency f_a. Think of this curve as the parabola in Fig. 11. Next, consider the same hull geometry containing an equipment of weight W_b and frequency f_b, subject to a charge weight Q_b. Equation (14) predicts the slope for system b. Multiplying both sides of eq. (14) by x yields the relationship between the ordinate values:

$$Y=y\left(\frac{Q_a}{Q_b}\right)^{(\frac{1}{3})}\left(\frac{W_a}{W_b}\right)^{(\frac{1}{6})}\left(\frac{f_b}{f_a}\right)^{(1.6)}=\alpha y \quad (33)$$

Y is the scaled ordinate through which a new parabola is passed and y is the corresponding point on the reference parabola. Selecting the two coordinates y_1 and y_2 from the reference parabola, we obtain the new ordinate values as

$$Y_1=\alpha y_1$$

$$Y_2=\alpha y_2$$

Y_1 and Y_2 and their corresponding shock factors are substituted into eqs. (29) and (30) to find the equation of the new parabola.

As an example of parabolic scaling for the same hulls, let the reference response data be obtained from model B, containing a 15-kip, 20-Hz equipment subject to a 1,160-lb charge. A parabola will be generated for model B, containing a 25-kip, 30-Hz equipment, subject to a 3,625-lb charge. Equation (33) yields

$$Y=1.52375y \quad (34)$$

The equation for the parabola that passes through the reference data that are listed in Appendix A is

$$y=40.7703x+21.6262x^2$$

Choosing $x_1 = 0.3$ and $x_2 = 0.6$, we obtain $y_1 = 14.1774$ and $y_2 = 32.2476$. Substituting these values into eq.(34) yields $Y_1 = 21.6029$ and $Y_2 = 49.1374$. These Y-coordinates are substituted into eqs.(31-32) to yield the scaled parabola

$$Y=62.1239x+32.9530x^2$$

which compares favorably with the actual parabola

$$Y=65.0874x+28.9880x^2$$

Figure 12 shows the plot of each curve. There is a very good closeness of fit between the scaled parabola (Y_{est}) and the parabola generated through the data (Y_{data}).

Different Hulls

The parabolic scaling procedure described above was developed for changes in the charge weight and the equipment assuming the same hull. It is now possible to extend this procedure to the case where either the existing equipment and charge weight or the new equipment and charge weight are assigned to a linearly scaled hull.

Let the least squares parabola through the data for the prototype be

$$y=Ex+Fx^2$$

This equation can be adjusted for the linearly scaled system as follows: let y_s be the peak acceleration and x_s be the shock factor in the scaled model. Recall that

$$y_s=\frac{y}{L};x_s=\sqrt{L}x \tag{35}$$

The equation of the parabola for the scaled system is

$$y_s=E_sx_s+F_sx_s^2 \tag{36}$$

where, by examining eqs.(29) and (30), it follows that

$$E_s=\frac{E}{(L^{(3/2)})};F_s=\frac{F}{L^2} \tag{37}$$

Thus, the coefficients E and F are scaled by eq.(37) to produce the scaled parabola represented by eq.(36).

Having the scaled parabolic equation in the linearly scaled system, we can now proceed to find the parabola for the conditions desired following the above described procedure for parabolic scaling for the same hull.

RESULTS

Linear Scaling

Table 13 summarizes the tests run on the validity of the general scaling rule given by eq.(17) for models B. The charge weights once again ranged from 600 lb to 3,625 lb; the equipment weight from 15 kips to 25 kips; and the frequency equal to 20 Hz and 30 Hz. In the case of model SBS, L = 26/33, while for model LBS, L = 40/33. The results show the predicted response slopes compared with the actual slopes for each configuration. The errors are generally less than one percent, except for the comparison between LBS model and model B subject to a charge weight equal to 1450 lb, where the largest error was 1.42%. Table 14 provides the same information for model F. Again, the errors are generally less that one percent, with the largest error being 1.16%

Equation(20) was similarly tested for models B and F. The results for model B are shown in Table 15. In the case of predicting the response slopes for model SB, the errors tend to be about 5% or less, except for the 15-kip, 20-Hz equipment where the error was 10.35%. The errors associated with model LB were less than 5% except in one case where it equaled 5.26%. The results for model F are listed in Table 16. The errors ranged from a low of 0.62% to a high of 7.68% for model SF. In the case of model LF the errors range from 0.36% to 12.13%.

A Word of Caution

Recall the time history motions in Fig.3 for the response of an equipment installed in model B, LBS, and SBS which was used to validate the general linear scaling law. Figure 13 shows the least square response slopes for models B, LB, and LBS. It is particularly important to observe that higher peak accelerations are experienced by the equipment placed in the linearly scaled hull, namely model LB, and lower peak accelerations in the case of the fully scaled model LBS. Consequently, if shock design values generated from model LBS were used without these scaling rules to predict the actual equipment in model B, the equipment would be underdesigned, and overdesigned if model LB were used. Just the opposite occurs for models SB and SBS as shown in Fig.14. Now the original equipment installed in Model B would be overdesigned if the predicted design values generated from model SB were used without the new scaling rules and underdesigned in the case of model SBS.

Parabolic Scaling

Two examples are presented for the parabolic scaling rules. In the first example, consider the least squares parabola through the data for model B, 15-kip and 20-Hz equipment, and a 1,160-lb charge:

$$y=40.7703x+21.6262x^2 \tag{38}$$

Let the linear scaled model by LBS, where L = 40/33. Thus, E_s = 30.5510 and F_s = 14.7193 in accord with eq.(37). The scaled parabola for model LBS, 26.713-kip, 16.5-Hz equipment, and 2,065.84-lb charge weight, is therefore,

$$y_s=30.5510x_s+14.7193x_s^2 \tag{39}$$

Suppose the equipment and charge weight are changed to W_b = 44.522 kips, f_b = 24.75 Hz, and Q_b = 12,911.48 lb. These conditions can be found in Table A5 so that the scaled parabola can be checked against the parabola based on the data. Choose two points on the parabola, i.e., x_{s1} = 0.330289 and x_{s2} = 0.660578. Eq.(39) yields y_{s1} = 11.6964 and y_{s2} = 26.6043. The scaling rule from eq.(14) yields α = 1.39729, so that the points through which the final scaled parabola must pass through Y_{s1} = 16.3433 and Y_{s2} = 37.1739. The equations (31) and (32) for E and F change to

$$E_s=\frac{(4Y_1-Y_2)}{(0.660578)}=42.6888$$

$$F_s=\frac{(Y_2-2Y_1)}{(0.218182)}=20.5668$$

since X_{s1} and x_{s2} are the abscissa values in the linearly scaled model. The final scaled parabola is

$$y_s=42.6888x_s+20.5668x_s^2 \tag{40}$$

This compares with the least squares parabola through the data in Table A5:

$$y_s=47.1556x_s+11.3756x_s^2 \tag{41}$$

Figure 15 shows a good fit between the two parabolas.

Models F and SFS were chosen for the second example of parabolic scaling. In this example the parabolic scaling rule is applied for the case where the equipment in model F remains unchanged when inserted into the linearly scaled model of the hull. The scaled parabola will be compared with the least squares parabola from the data in Table B2 for model SF. The least squares parabola for model F with a 15-kip, 30-Hz equipment, and a 900-lb charge is

$$y=100.2522x+35.3602x^2 \tag{42}$$

The scaling factor L = 30.83/40.3, so that the scaled parabola for the SFS model is

$$y_s=143.3530x_s+60.4186x_s^2 \tag{43}$$

where, from Table B3, Q_a = 440.17 lb, W_a = 6.715 kips, and f_a = 39.216 Hz. Since Q_b = 900 lb, W_b = 15 kips, and f_b = 30 Hz, the linear scaling α = 0.521016. In this example, substitute x_{s1} = 0.3 and x_{s2} = 0.6 into eq.(43), so that y_{s1} = 25.239855 and y_{s2} = 56.145938. Thus,

$$E=\frac{(4Y_{s1}-Y_{s2})}{(0.6)}=74.6891$$

$$F=\frac{(Y_{s2}-2Y_{s1})}{(0.18)}=31.4790$$

The final scaled parabola is

$$y_s=74.6891x_s+31.4790x_s^2 \tag{44}$$

This compares with the least squares parabola through the data in Table B2:

$$y_s=86.9641x_s+15.3309x_s^2 \tag{45}$$

The two parabolas shown good agreement as seen in Fig.16.

SUMMARY AND CONCLUSIONS

The results of this study have demonstrated that useful information may be obtained by using a computer as an initial surrogate for shock testing purposes. These results show the relative changes in shock design values for different boats and attack geometries. It is emphasized that the test sections were small in size and devoid of typical equipment present in a real compartment. Consequently, the results provide only trends in shock design values rather than absolute design numbers.

The general linear scaling law that has been used in practice was shown to provide excellent results for the time history responses generated by the HULL code. One problem is, of course, how to predict the equipment peak accelerations for the case where the hull is fixed and the charge weight, equipment weight, and/or the equipment fixed base frequency varies. The second problem is how to predict these peak accelerations when the equipment and/or charge size are not varied, or a new equipment is installed in the scaled hull.

An attempt to answer the first problem required the collection of large amounts of computer generated data for two submarine models, each of which contained a single-degree of freedom frame-mounted equipment. The recommended rules for scaling on charge weight, equipment weight, and equipment fixed base frequency for a fixed hull size provides a reasonably wide numerical range. The range of errors associated with the scaling rule of eq.(14) are within engineering accuracy. The intent of this general scaling rule is to allow greater usage of existing shock response data for different equipment subject to a variety of shock conditions.

The answer to the second and major problem was answered in two steps. The first step, represented by eq.(17), showed how to obtain the response for the fully scaled model from the unscaled model. The second step, represented by eq.(18), produced a scaling rule that allows the charge weight, equipment weight, and/or the equipment frequency to change to either the original values, in which case eq.(20) is used, or to an entirely new equipment that might be installed in the scaled hull or a new charge weight.

It was shown that care must be exercised when shock design values are generated from a fully scaled model or a scaled hull containing the same equipment as the unscaled model. Depending on the circumstances the generated shock design values may be either too high or too low if the scaling rules are ignored.

A parabolic least squares fit through the data is better than a straight line fit. It was demonstrated that if a parabola is known for a prototype model, a scaled parabola can be generated for the case where the same equipment and charge weight apply for a linearly scaled model or a different equipment and/or charge weight is used in the scaled model. The two example problems showed rather good agreement between the scaled parabola and the least squares parabola generated through the data.

ACKNOWLEDGEMENTS

This study is part of a project supported by the Office of Naval Research. The content of this report does not necessarily reflect the position or policy of the government.

Robert Pohland and Robert Wagner, mechanical engineering graduate students, are recognized for their assistance with this project.

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Table 1 - Equipment Weight Scaling

MODEL B

15 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.5073	6.0219	5.4830
0.30	13.9880	13.2670	12.6493
0.45	23.0325	22.2268	18.9597
0.60	32.1668	30.3143	27.3157
0.75	42.7179	39.4709	36.0714
Slopes	54.0409	50.6482	45.7309

20 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.2250	5.7879	5.3364
0.30	13.6857	12.9770	12.0856
0.45	22.0331	21.3429	18.5255
0.60	30.6274	29.3813	26.1935
0.75	40.8301	37.5882	34.3608
Slopes	51.6795	48.6348	43.8378

25 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.0044	5.6625	5.3142
0.30	13.3875	12.6960	11.5630
0.45	21.9438	20.8516	18.1265
0.60	29.1889	28.5723	25.5814
0.75	38.9767	35.8472	33.5140
Slopes	49.7273	46.9253	42.7534

30 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	5.8753	5.5372	5.1883
0.30	13.0928	12.4159	11.1524
0.45	21.4400	20.3693	17.7374
0.60	27.8608	28.1286	24.9957
0.75	37.1996	34.1760	32.7573
Slopes	47.7360	45.4389	41.7545

35 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	5.7469	5.4161	5.0975
0.30	12.8060	12.1447	10.9148
0.45	20.9731	19.9087	17.3535
0.60	26.8360	27.4854	24.4649
0.75	35.6445	32.8141	32.0025
Slopes	46.0418	44.0538	40.8315

15 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	12.8666	12.1329	10.8451
0.30	28.6857	27.2055	24.4456
0.45	46.9695	44.5929	38.8641
0.60	60.0099	61.0938	54.7966
0.75	79.7418	73.4683	71.6140
Slopes	103.0176	98.4290	91.3436

20 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	12.2366	11.5385	10.7133
0.30	27.2771	25.8732	23.2793
0.45	44.7531	42.4929	36.9790
0.60	56.8628	58.2238	52.1899
0.75	75.5542	69.6130	68.2240
Slopes	97.7300	93.5423	87.0410

25 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	11.6715	11.0003	10.1368
0.30	26.0330	24.6671	22.1540
0.45	42.6144	40.4741	35.2021
0.60	54.0933	55.4650	49.6744
0.75	71.6469	66.1916	64.9949
Slopes	92.8713	89.0394	82.8755

30 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	11.1481	10.4986	9.7676
0.30	24.8508	23.5437	21.1498
0.45	40.5810	38.5338	33.6305
0.60	51.4935	53.2217	47.2796
0.75	67.9178	62.9388	61.9326
Slopes	88.2613	84.9416	78.9988

35 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	10.6397	10.0192	9.4202
0.30	23.7144	22.4688	20.2095
0.45	38.6907	36.7107	32.1096
0.60	49.0343	50.6937	45.1784
0.75	64.6361	59.8678	58.9606
Slopes	84.0555	80.8730	75.3557

Table 2 - n Values for Scaling on All Weight Combinations: Model B

20 Hz

Weight Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	0.1891	0.1646	0.1337	0.1625
30/15	2.0000	0.1790	0.1566	0.1312	0.1556
35/20	1.7500	0.2064	0.1768	0.1269	0.1700
25/15	1.6667	0.1628	0.1495	0.1318	0.1480
30/20	1.5000	0.1958	0.1676	0.1201	0.1612
35/25	1.4000	0.2289	0.1877	0.1367	0.1844
20/15	1.3333	0.1553	0.1410	0.1470	0.1478
25/20	1.2500	0.1726	0.1604	0.1122	0.1484
30/25	1.2000	0.2242	0.1765	0.1297	0.1768
35/30	1.1667	0.2344	0.2008	0.1450	0.1934
	Average	0.1948	0.1681	0.1314	
	Std. Dev	0.0266	0.0170	0.0099	
	max	0.2344	0.2008	0.1470	
	min	0.1553	0.1410	0.1122	
		overall average		0.1648	
		overall max		0.2344	
		overall min		0.1122	

30 Hz

Weight Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	0.2401	0.2317	0.2271	0.2330
30/15	2.0000	0.2230	0.2125	0.2095	0.2150
35/20	1.7500	0.2693	0.2601	0.2576	0.2623
25/15	1.6667	0.2030	0.1961	0.1905	0.1965
30/20	1.5000	0.2513	0.2379	0.2391	0.2428
35/25	1.4000	0.2964	0.2859	0.2827	0.2883
20/15	1.3333	0.1832	0.1767	0.1677	0.1758
25/20	1.2500	0.2285	0.2211	0.2198	0.2231
30/25	1.2000	0.2792	0.2584	0.2628	0.2668
35/30	1.1667	0.3167	0.3184	0.3063	0.3138
	Average	0.2491	0.2399	0.2363	
	Std. Dev	0.0397	0.0402	0.0401	
	max	0.3167	0.3184	0.3063	
	min	0.1832	0.1767	0.1677	
		overall average		0.2417	
		overall max		0.3184	
		overall min		0.1677	

Table 3 - Scaled Slopes for Model B
Scaled on 25 kip Using n=1/6

20 Hz

	1160#	%error	1740#	%error	3625#	%error
15k	49.6304	0.19	46.5146	0.88	41.9986	1.77
20k	49.7928	0.13	46.8593	0.14	42.2374	1.21
25k	49.7273	0.00	46.9253	0.00	42.7534	0.00
30k	49.2088	1.04	46.8408	0.18	43.0428	0.68
35k	48.6975	2.07	46.5949	0.70	43.1867	1.01

30 Hz

	1160#	%error	1740#	%error	3625#	%error
15k	94.6099	1.87	90.3958	1.52	83.8887	1.22
20k	94.1621	1.39	90.1273	1.22	83.8633	1.19
25k	92.8713	0.00	89.0394	0.00	82.8755	0.00
30k	90.9845	2.03	87.5623	1.66	81.4362	1.74
35k	88.9039	4.27	85.5378	3.93	79.7023	3.83

Table 4 - Equipment Weight Scaling

MODEL F

15 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	7.6863	7.1291	6.1514
0.30	15.8125	15.6288	15.1816
0.45	26.9363	26.5787	22.6295
0.60	37.6583	35.8768	32.4759
0.75	49.2544	46.6812	43.0371
Slopes	62.6697	60.0043	54.4839

15 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	15.1687	14.3238	12.7859
0.30	33.2585	31.6423	29.1711
0.45	52.6217	52.0236	45.4052
0.60	70.3431	67.3839	63.9906
0.75	92.5846	87.0836	83.9613
Slopes	119.2541	113.7737	107.0440

20 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	7.4226	6.8157	6.0474
0.30	15.2183	15.0171	14.7941
0.45	25.9723	25.4454	21.7966
0.60	36.2894	34.4744	31.3103
0.75	47.4665	44.8990	41.5190
Slopes	60.3959	57.6459	52.5893

20 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	14.5925	13.7703	12.2963
0.30	31.9543	30.4066	27.9583
0.45	50.6323	50.0847	43.6672
0.60	66.7460	63.6517	61.5868
0.75	88.6260	81.7697	80.6869
Slopes	114.0015	107.6719	102.9086

25 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	7.1653	6.6597	5.9444
0.30	15.4732	14.7190	14.3061
0.45	25.0337	24.5306	21.1075
0.60	34.9587	33.2822	30.1752
0.75	45.7280	43.2724	40.0407
Slopes	58.3864	55.6582	50.7616

25 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	14.0275	13.2726	11.8306
0.30	30.7098	29.2302	26.9124
0.45	48.6845	48.2118	41.9781
0.60	64.1160	60.7416	59.2508
0.75	84.9972	78.6681	77.6632
Slopes	109.4486	103.3546	99.0193

30 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.9336	6.5440	5.8438
0.30	15.2025	14.4612	13.8355
0.45	24.1346	23.8359	20.7502
0.60	33.7072	32.3773	29.2618
0.75	44.0465	41.6805	38.6290
Slopes	56.3398	53.9256	49.2070

30 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	13.5212	12.8045	11.4184
0.30	29.6098	28.1280	25.9079
0.45	46.8287	46.3989	40.3559
0.60	61.6885	58.9660	57.0072
0.75	81.4811	75.6047	74.8017
Slopes	105.1377	99.6539	95.3138

35 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.8157	6.4325	5.7441
0.30	14.9391	14.2119	13.3835
0.45	23.6704	23.3988	20.3966
0.60	32.4637	31.2470	28.7386
0.75	42.4228	40.1564	37.7670
Slopes	54.5059	52.2209	48.1806

35 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	13.0466	12.3554	11.0171
0.30	28.5474	27.1222	24.9319
0.45	45.0186	44.6441	38.8914
0.60	59.4417	56.8447	54.8269
0.75	78.3531	72.7447	71.9997
Slopes	101.1793	95.9557	91.7408

Table 5 - n Values for Scaling on All Weight Combinations: Model F

20 Hz

Weight Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	0.1647	0.1640	0.1451	0.1579
30/15	2.0000	0.1536	0.1541	0.1470	0.1516
35/20	1.7500	0.1834	0.1766	0.1565	0.1721
25/15	1.6667	0.1386	0.1472	0.1385	0.1414
30/20	1.5000	0.1715	0.1645	0.1640	0.1666
35/25	1.4000	0.2044	0.1895	0.1551	0.1830
20/15	1.3333	0.1285	0.1394	0.1230	0.1303
25/20	1.2500	0.1516	0.1573	0.1585	0.1558
30/25	1.2000	0.1957	0.1735	0.1706	0.1799
35/30	1.1667	0.2147	0.2084	0.1367	0.1866
	Average	0.1707	0.1674	0.1495	
	Std. Dev	0.0271	0.0194	0.0135	
	max	0.2147	0.2084	0.1706	
	min	0.1285	0.1394	0.1230	
		overall average		0.1625	
		overall max		0.2147	
		overall min		0.1230	

30 Hz

Weight Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	0.1940	0.2010	0.1821	0.1924
30/15	2.0000	0.1818	0.1912	0.1674	0.1801
35/20	1.7500	0.2132	0.2059	0.2053	0.2081
25/15	1.6667	0.1680	0.1880	0.1525	0.1695
30/20	1.5000	0.1996	0.1909	0.1891	0.1932
35/25	1.4000	0.2335	0.2208	0.2269	0.2270
20/15	1.3333	0.1566	0.1916	0.1370	0.1617
25/20	1.2500	0.1826	0.1834	0.1727	0.1796
30/25	1.2000	0.2204	0.2000	0.2092	0.2099
35/30	1.1667	0.2490	0.2453	0.2479	0.2474
	Average	0.1999	0.2018	0.1890	
	Std. Dev	0.0277	0.0177	0.0322	
	max	0.2490	0.2453	0.2479	
	min	0.1566	0.1834	0.1370	
		overall average		0.1969	
		overall max		0.2490	
		overall min		0.1370	

Table 6 - Scaled Slopes for Model F
Scaled on 25 kip Using n=1/6

20 Hz

	1160#	%error	1740#	%error	3625#	%error
15k	57.5550	1.42	55.1071	0.99	50.0372	1.43
20k	58.1910	0.33	55.5414	0.21	50.6694	0.18
25k	58.3864	0.00	55.6582	0.00	50.7616	0.00
30k	58.0781	0.53	55.5894	0.12	50.7252	0.07
35k	57.6499	1.26	55.2330	0.76	50.9597	0.39

30 Hz

	1160#	%error	1740#	%error	3625#	%error
15k	109.5213	0.07	104.4882	1.10	98.3077	0.72
20k	109.8396	0.36	103.7411	0.37	99.1517	0.13
25k	109.4486	0.00	103.3546	0.00	99.0193	0.00
30k	108.3815	0.97	102.7286	0.61	98.2545	0.77
35k	107.0154	2.22	101.4905	1.80	97.0325	2.01

Table 7 - Equipment Frequency Scaling

MODEL B

15 kip, 15 Hz

SF	1160#	1740#	3625#
0.15	3.9208	3.6317	3.3123
0.30	8.2642	8.0054	7.6219
0.45	13.9582	13.2208	11.3417
0.60	19.5099	18.4967	16.5564
0.75	25.8466	23.9049	21.7263
Slopes	32.6784	30.6444	27.5683

15 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.5073	6.0219	5.4830
0.30	13.9880	13.2670	12.6493
0.45	23.0325	22.2268	18.9597
0.60	32.1668	30.3143	27.3157
0.75	42.7179	39.4709	36.0714
Slopes	54.0409	50.6482	45.7309

15 kip, 25 Hz

SF	1160#	1740#	3625#
0.15	9.4058	8.8646	8.0548
0.30	20.9620	19.8770	18.1005
0.45	34.3768	32.6577	28.3904
0.60	45.7042	45.0957	40.0706
0.75	60.9737	56.0543	52.5091
Slopes	77.8358	73.6055	66.9400

15 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	12.8666	12.1329	10.8451
0.30	28.6857	27.2055	24.4456
0.45	46.9695	44.5929	38.8641
0.60	60.0099	61.0938	54.7966
0.75	79.7418	73.4683	71.6140
Slopes	103.0176	98.4290	91.3436

15 kip, 35 Hz

SF	1160#	1740#	3625#
0.15	16.4750	15.5262	14.1614
0.30	36.7314	34.8182	31.3474
0.45	60.2808	57.2295	49.7863
0.60	76.6229	79.0245	70.2822
0.75	101.6191	93.6162	91.9194
Slopes	131.5597	126.1855	117.2049

20 kip, 15 Hz

SF	1160#	1740#	3625#
0.15	3.8200	3.5368	3.2634
0.30	8.0329	7.7929	7.4322
0.45	13.5841	13.0590	11.0401
0.60	18.9871	18.0067	16.1112
0.75	25.1629	23.2663	21.1484
Slopes	31.8062	29.8979	26.8406

20 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.2250	5.7857	5.3364
0.30	13.6857	12.9727	12.0856
0.45	22.0331	21.3429	18.5255
0.60	30.6274	29.6133	26.1935
0.75	40.8301	37.5730	34.3608
Slopes	51.6795	48.7367	43.8378

20 kip, 25 Hz

SF	1160#	1740#	3625#
0.15	9.0876	8.5646	7.8454
0.30	20.2512	19.2047	17.2548
0.45	33.1516	31.4834	27.4390
0.60	42.4712	43.4779	38.6750
0.75	56.6716	52.0384	50.6362
Slopes	73.0046	69.7609	64.5518

20 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	12.2388	11.5340	10.7133
0.30	27.2689	25.8647	23.2793
0.45	44.7597	42.4929	36.9790
0.60	56.9071	58.6734	52.1899
0.75	75.5359	69.5853	68.2240
Slopes	97.7410	93.7409	87.0410

20 kip, 35 Hz

SF	1160#	1740#	3625#
0.15	15.4735	14.5724	13.2369
0.30	34.4956	32.6803	29.3425
0.45	56.3908	53.5439	46.6737
0.60	71.5830	73.9493	65.7072
0.75	94.5249	87.4860	86.0448
Slopes	122.7387	118.0354	109.6964

(Table 7 con't.)

25 kip, 15 Hz

SF	1160#	1740#	3625#
0.15	3.7212	3.4475	3.2312
0.30	7.8153	7.5849	7.2463
0.45	13.2181	12.7537	10.7545
0.60	18.4763	17.5273	15.6757
0.75	24.4946	22.6423	20.6980
Slopes	30.9557	29.1151	26.2036

25 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	6.0044	5.6625	5.3142
0.30	13.3875	12.6960	11.5630
0.45	21.9438	20.8516	18.1265
0.60	29.1889	28.5723	25.5814
0.75	38.9767	35.8472	33.5140
Slopes	49.7273	46.9253	42.7534

25 kip, 25 Hz

SF	1160#	1740#	3625#
0.15	8.7781	8.2727	7.7059
0.30	19.5595	18.5506	16.6819
0.45	32.0629	30.4370	26.5131
0.60	40.9129	42.0232	37.3947
0.75	54.2584	50.0335	48.8693
Slopes	70.1853	67.2661	62.3678

25 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	11.6715	11.0003	10.1368
0.30	26.0330	24.6671	22.1540
0.45	42.6144	40.4741	35.2021
0.60	54.0933	55.4650	49.6744
0.75	71.6469	66.1916	64.9949
Slopes	92.8713	89.0394	82.8755

25 kip, 35 Hz

SF	1160#	1740#	3625#
0.15	14.5222	13.6753	12.5167
0.30	32.3692	30.6690	27.5831
0.45	52.8067	50.1044	43.8278
0.60	66.9270	69.1888	61.6636
0.75	88.2231	81.7136	80.4795
Slopes	114.7278	110.3818	102.8143

Table 8 - n Values for Scaling on All Frequency Combinations: Model B

15 kip

Frequency Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.6438	1.6704	1.7081	1.6741
30/15	2.0000	1.6565	1.6835	1.7283	1.6894
35/20	1.7500	1.5899	1.6312	1.6818	1.6343
25/15	1.6667	1.6990	1.7154	1.7367	1.7170
30/20	1.5000	1.5912	1.6387	1.7063	1.6454
35/25	1.4000	1.5599	1.6020	1.6647	1.6089
20/15	1.3333	1.7486	1.7466	1.7593	1.7515
25/20	1.2500	1.6351	1.6752	1.7075	1.6726
30/25	1.2000	1.5374	1.5940	1.7049	1.6121
35/30	1.1667	1.5865	1.6115	1.6172	1.6051
	Average	1.6248	1.6568	1.7015	
	Std. Dev	0.0616	0.0477	0.0377	
	max	1.7486	1.7466	1.7593	
	min	1.5374	1.5940	1.6172	

overall average	1.6610
overall max	1.7593
overall min	1.5374

20 kip

Frequency Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.5938	1.6207	1.6615	1.6253
30/15	2.0000	1.6197	1.6486	1.6973	1.6552
35/20	1.7500	1.5457	1.5806	1.6390	1.5885
25/15	1.6667	1.6265	1.6587	1.7179	1.6677
30/20	1.5000	1.5717	1.6132	1.6916	1.6255
35/25	1.4000	1.5441	1.5630	1.5759	1.5610
20/15	1.3333	1.6873	1.6986	1.7053	1.6970
25/20	1.2500	1.5482	1.6072	1.7342	1.6299
30/25	1.2000	1.6005	1.6205	1.6395	1.6202
35/30	1.1667	1.4774	1.4950	1.5007	1.4910
	Average	1.5815	1.6106	1.6563	
	Std. Dev	0.0547	0.0531	0.0682	
	max	1.6873	1.6986	1.7342	
	min	1.4774	1.4950	1.5007	

overall average	1.6161
overall max	1.7342
overall min	1.4774

(Table 8 con't.)

Frequency Ratio	Decimal Ratio	25 kip 1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.5461	1.5729	1.6134	1.5775
30/15	2.0000	1.5850	1.6127	1.6612	1.6196
35/20	1.7500	1.4939	1.5285	1.5680	1.5301
25/15	1.6667	1.6025	1.6393	1.6976	1.6464
30/20	1.5000	1.5406	1.5797	1.6324	1.5842
35/25	1.4000	1.4605	1.4720	1.4856	1.4727
20/15	1.3333	1.6476	1.6591	1.7017	1.6695
25/20	1.2500	1.5442	1.6138	1.6922	1.6167
30/25	1.2000	1.5362	1.5381	1.5593	1.5445
35/30	1.1667	1.3710	1.3939	1.3985	1.3878
	Average	1.5328	1.5610	1.6010	
	Std. Dev	0.0735	0.0769	0.0947	
	max	1.6476	1.6591	1.7017	
	min	1.3710	1.3939	1.3985	

overall average	1.5649
overall max	1.7017
overall min	1.3710

Table 9 - Scaled Slopes for Model B
Scaled on 25 Hz Using n=1.6

	1160#	%error	1740#	%error	3625#	%error
15 kip						
15 Hz	73.9978	4.93	69.3920	5.72	62.4264	6.74
20 Hz	77.2287	0.78	72.3802	1.66	65.3530	2.37
25 Hz	77.8358	0.00	73.6055	0.00	66.9400	0.00
30 Hz	76.9523	1.14	73.5247	0.11	68.2320	1.93
35 Hz	76.7924	1.34	73.6554	0.07	68.4134	2.20
20 kip						
15 Hz	72.0228	1.34	67.7016	2.95	60.7785	5.85
20 Hz	73.8541	1.16	69.6486	0.16	62.6476	2.95
25 Hz	73.0046	0.00	69.7609	0.00	64.5518	0.00
30 Hz	73.0107	0.01	70.0227	0.38	65.0180	0.72
35 Hz	71.6435	1.86	68.8981	1.24	64.0306	0.81
25 kip						
15 Hz	70.0969	0.13	65.9290	1.99	59.3361	4.86
20 Hz	71.0642	1.25	67.0599	0.31	61.0980	2.04
25 Hz	70.1853	0.00	67.2661	0.00	62.3678	0.00
30 Hz	69.3732	1.16	66.5108	1.12	61.9065	0.74
35 Hz	66.9675	4.58	64.4307	4.22	60.0135	3.77

Table 10 - Equipment Frequency Scaling

MODEL F

15 kip, 15 Hz

SF	1160#	1740#	3625#
0.15	4.5888	4.2022	3.6402
0.30	9.4939	9.3079	9.0414
0.45	16.1761	15.7945	13.5711
0.60	22.6121	21.5873	19.4440
0.75	29.5111	27.9414	25.7441
Slopes	37.5890	35.9100	32.5979

15 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	7.6863	7.1291	6.1514
0.30	15.8125	15.6288	15.1816
0.45	26.9363	26.5787	22.6295
0.60	37.6583	35.8768	32.4759
0.75	49.2544	46.6812	43.0371
Slopes	62.6697	60.0044	54.4839

15 kip, 25 Hz

SF	1160#	1740#	3625#
0.15	11.1139	10.3878	9.2764
0.30	24.1389	22.9602	22.1982
0.45	38.7966	38.0097	32.9400
0.60	54.1848	51.9961	46.7595
0.75	70.8159	66.9909	62.0838
Slopes	90.4970	86.4578	78.7818

15 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	15.1687	14.3238	12.7859
0.30	33.2585	31.6423	29.1711
0.45	52.6217	52.0236	45.4052
0.60	70.3431	67.3839	63.9906
0.75	92.5846	87.0836	83.9613
Slopes	119.2541	113.7737	107.0440

15 kip, 35 Hz

SF	1160#	1740#	3625#
0.15	19.6010	18.4962	16.5161
0.30	42.9113	40.8360	37.5713
0.45	68.0186	67.3040	58.6563
0.60	89.6583	85.5058	82.7505
0.75	118.9112	109.8490	108.4437
Slopes	153.0508	144.6483	138.2846

20 kip, 15 Hz

SF	1160#	1740#	3625#
0.15	4.5008	4.1203	3.5705
0.30	9.2948	9.1131	8.8727
0.45	15.8356	15.4791	13.2923
0.60	22.1322	21.1494	19.0538
0.75	28.9116	27.3473	25.2363
Slopes	36.8101	35.1658	31.9502

20 kip, 20 Hz

SF	1160#	1740#	3625#
0.15	7.4226	6.8157	6.0474
0.30	15.2183	15.0171	14.7941
0.45	25.9723	25.4454	21.7966
0.60	36.2894	34.4744	31.3103
0.75	47.4665	44.8990	41.5190
Slopes	60.3959	57.6459	52.5893

20 kip, 25 Hz

SF	1160#	1740#	3625#
0.15	10.7150	10.1127	9.0306
0.30	23.4892	22.3452	21.0681
0.45	37.2288	36.7982	32.0676
0.60	51.0868	49.1826	45.1785
0.75	66.7710	63.1937	59.3924
Slopes	85.7675	82.1693	75.7631

20 kip, 30 Hz

SF	1160#	1740#	3625#
0.15	14.0275	13.7757	11.8305
0.30	30.7098	30.4166	26.9123
0.45	48.6845	50.0847	41.9781
0.60	64.1160	63.1655	59.2507
0.75	84.9971	81.8023	77.6631
Slopes	109.4485	107.4590	99.0191

20 kip, 35 Hz

SF	1160#	1740#	3625#
0.15	18.6050	17.6189	15.7121
0.30	40.7544	38.7136	35.6700
0.45	64.4914	63.8860	55.5857
0.60	84.9684	81.0488	78.5035
0.75	112.2862	104.1319	102.9860
Slopes	144.8355	137.1587	131.2428

(Table 10 con't.)

25 kip, 15 Hz			
SF	1160#	1740#	3625#
0.15	4.4143	4.0433	3.5018
0.30	9.0994	8.9216	8.7088
0.45	15.5011	15.1690	13.0182
0.60	21.6806	20.7193	18.6704
0.75	28.3226	26.7777	24.7370
Slopes	36.0548	34.4436	31.3140

25 kip, 20 Hz			
SF	1160#	1740#	3625#
0.15	7.1653	6.6597	5.9444
0.30	15.4732	14.7190	14.3061
0.45	25.0337	24.5306	21.1075
0.60	34.9587	33.2822	30.1752
0.75	45.7280	43.2724	40.0407
Slopes	58.3864	55.6582	50.7616

25 kip, 25 Hz			
SF	1160#	1740#	3625#
0.15	10.4308	9.8438	8.7903
0.30	22.8540	21.7438	19.9804
0.45	36.1851	35.7715	31.2146
0.60	48.1966	46.5010	43.9989
0.75	63.5570	59.5971	57.7095
Slopes	81.8504	78.1376	73.5682

25 kip, 30 Hz			
SF	1160#	1740#	3625#
0.15	14.0275	13.2726	11.8306
0.30	30.7098	29.2302	26.9124
0.45	48.6845	48.2118	41.9781
0.60	64.1160	60.7416	59.2508
0.75	84.9972	78.6681	77.6632
Slopes	109.4486	103.3546	99.0193

25 kip, 35 Hz			
SF	1160#	1740#	3625#
0.15	17.7210	16.7824	14.9644
0.30	38.7754	36.8400	33.8630
0.45	61.1187	60.6164	52.8287
0.60	80.7443	77.2179	74.4411
0.75	106.3868	98.8174	97.7651
Slopes	137.3987	130.3358	124.5777

25 kip, 40 Hz			
SF	1160#	1740#	3625#
0.15	21.2398	20.1160	17.9333
0.30	46.3784	44.0780	40.6902
0.45	72.8766	72.3270	63.2746
0.60	96.7689	92.5348	89.0558
0.75	126.7121	118.4159	116.4610
Slopes	164.0318	156.0572	148.8079

Table 11 - n Values for Scaling on All Frequency Combinations: Model F

15 kip

Frequency Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.6571	1.6444	1.7055	1.6690
30/15	2.0000	1.6657	1.6637	1.7154	1.6816
35/20	1.7500	1.5955	1.5723	1.6644	1.6107
25/15	1.6667	1.7200	1.7200	1.7275	1.7225
30/20	1.5000	1.5868	1.5779	1.6656	1.6101
35/25	1.4000	1.5617	1.5295	1.6721	1.5878
20/15	1.3333	1.7768	1.7846	1.7855	1.7823
25/20	1.2500	1.6466	1.6368	1.6526	1.6454
30/25	1.2000	1.5135	1.5059	1.6814	1.5669
35/30	1.1667	1.6186	1.5575	1.6612	1.6125
	Average	1.6342	1.6193	1.6931	
	Std. Dev	0.0729	0.0831	0.0390	
	max	1.7768	1.7846	1.7855	
	min	1.5135	1.5059	1.6526	

overall average	1.6489
overall max	1.7855
overall min	1.5059

20 kip

Frequency Ratio	Decimal Ratio	1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.6167	1.6064	1.6675	1.6302
30/15	2.0000	1.5721	1.6115	1.6319	1.6052
35/20	1.7500	1.5630	1.5490	1.6342	1.5821
25/15	1.6667	1.6559	1.6614	1.6903	1.6692
30/20	1.5000	1.4663	1.5360	1.5607	1.5210
35/25	1.4000	1.5572	1.5227	1.6329	1.5710
20/15	1.3333	1.7212	1.7180	1.7322	1.7238
25/20	1.2500	1.5717	1.5885	1.6362	1.5988
30/25	1.2000	1.3373	1.4717	1.4683	1.4258
35/30	1.1667	1.8173	1.5831	1.8277	1.7427
	Average	1.5879	1.5848	1.6482	
	Std. Dev	0.1247	0.0672	0.0907	
	max	1.8173	1.7180	1.8277	
	min	1.3373	1.4717	1.4683	

overall average	1.6070
overall max	1.8277
overall min	1.3373

(Table 11 con't.)

Frequency Ratio	Decimal Ratio	25 kip 1160# n	1740# n	3625# n	Average n
35/15	2.3333	1.5790	1.5706	1.6297	1.5931
30/15	2.0000	1.6020	1.5853	1.6609	1.6161
35/20	1.7500	1.5293	1.5205	1.6043	1.5513
25/15	1.6667	1.6050	1.6036	1.6721	1.6269
30/20	1.5000	1.5498	1.5265	1.6479	1.5747
35/25	1.4000	1.5395	1.5206	1.5654	1.5418
20/15	1.3333	1.6756	1.6682	1.6792	1.6743
25/20	1.2500	1.5139	1.5203	1.6629	1.5657
30/25	1.2000	1.5937	1.5341	1.6295	1.5858
35/30	1.1667	1.4754	1.5047	1.4895	1.4899
	Average	1.5663	1.5554	1.6242	
	Std. Dev	0.0539	0.0487	0.0555	
	max	1.6756	1.6682	1.6792	
	min	1.4754	1.5047	1.4895	

overall average	1.5820
overall max	1.6792
overall min	1.4754

Table 12 - Scaled Slopes for Model F
Scaled on 25 Hz Using n=1.6

15 kip

	1160#	%error	1740#	%error	3625#	%error
15 Hz	85.1175	5.94	81.3155	5.95	73.8155	6.30
20 Hz	89.5599	1.04	85.7510	0.82	77.8618	1.17
25 Hz	90.4970	0.00	86.4578	0.00	78.7818	0.00
30 Hz	89.0806	1.57	84.9869	1.70	79.9599	1.50
35 Hz	89.3369	1.28	84.4323	2.34	80.7177	2.46
40 Hz	88.5025	2.20	83.8507	3.02	80.2410	1.85

20 kip

	1160#	%error	1740#	%error	3625#	%error
15 Hz	83.3537	2.81	79.6303	3.09	72.3488	4.51
20 Hz	86.3105	0.63	82.3805	0.26	75.1542	0.80
25 Hz	85.7675	0.00	82.1693	0.00	75.7631	0.00
30 Hz	81.7560	4.68	80.2699	2.31	73.9655	2.37
35 Hz	84.5415	1.43	80.0605	2.57	76.6074	1.11
40 Hz	82.6863	3.59	78.5198	4.44	74.9684	1.05

25 kip

	1160#	%error	1740#	%error	3625#	%error
15 Hz	81.6434	0.25	77.9950	0.18	70.9082	3.62
20 Hz	83.4387	1.94	79.5399	1.79	72.5423	1.39
25 Hz	81.8504	0.00	78.1376	0.00	73.5682	0.00
30 Hz	81.7561	0.12	77.2040	1.19	73.9656	0.54
35 Hz	80.2006	2.02	76.0780	2.64	72.7169	1.16
40 Hz	77.3279	5.53	73.5685	5.85	70.1510	4.64

Table 13 - Application of General
Scaling Law, Eq. (17): Model B

Model B -- Predicted Slopes for Model SBS

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	81.1283	78.6741	77.2740	74.8540	72.4228	65.3914
15k,30Hz	152.9645	150.5211	147.3066	143.1768	140.7453	130.6138
20k,20Hz	77.4867	75.0109	73.8974	71.7688	69.5437	62.6845
20k,30Hz	145.1659	142.8177	139.7457	135.9652	133.7577	124.4614
25k,20Hz	74.0050	71.9499	71.1059	68.8254	67.0994	61.1338
25k,30Hz	137.6053	135.4997	132.7982	129.2238	127.3189	118.5051

Model SBS -- Actual Slopes

15k,20Hz	80.9118	78.5517	77.1814	73.8698	72.5807	65.4083
15k,30Hz	152.6595	150.2878	147.3027	143.9568	141.0518	130.6443
20k,20Hz	77.2505	74.8928	73.8961	70.8851	69.6949	62.7000
20k,30Hz	144.8728	142.5926	139.7395	136.7946	134.0465	124.4877
25k,20Hz	73.8018	71.8525	71.1154	68.7721	67.2440	61.1555
25k,30Hz	137.3274	135.2861	132.8187	130.2371	127.5945	118.5527

% Errors

15k,20Hz	0.27	0.16	0.12	1.33	0.22	0.03
15k,30Hz	0.20	0.16	0.00	0.54	0.22	0.02
20k,20Hz	0.31	0.16	0.00	1.25	0.22	0.02
20k,30Hz	0.20	0.16	0.00	0.61	0.22	0.02
25k,20Hz	0.28	0.14	0.01	0.08	0.22	0.04
25k,30Hz	0.20	0.16	0.02	0.78	0.22	0.04

Model B -- Predicted Slopes for Model LBS

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	42.5150	41.2289	40.4952	39.2270	37.9529	34.2681
15k,30Hz	80.1606	78.8801	77.1956	75.0313	73.7571	68.4478
20k,20Hz	40.6066	39.3092	38.7257	37.6102	36.4442	32.8496
20k,30Hz	76.0737	74.8432	73.2333	71.2522	70.0953	65.2236
25k,20Hz	38.7821	37.7051	37.2628	36.0677	35.1632	32.0370
25k,30Hz	72.1116	71.0082	69.5925	67.7194	66.7211	62.1022

Model LBS -- Actual Slopes

15k,20Hz	42.4081	41.1729	40.5210	38.7174	38.0260	34.2675
15k,30Hz	80.0119	78.7736	77.2320	73.9772	73.9001	68.4459
20k,20Hz	40.5034	39.2558	38.7454	37.1798	36.5150	32.8497
20k,30Hz	75.9329	74.7424	73.2678	70.2559	70.2320	65.2219
25k,20Hz	38.6829	37.6626	37.2876	35.6633	35.2302	32.0390
25k,30Hz	71.9790	70.9133	69.6398	66.8721	66.8517	62.1131

% Errors

15k,20Hz	0.25	0.14	0.06	1.32	0.19	0.00
15k,30Hz	0.19	0.14	0.05	1.42	0.19	0.00
20k,20Hz	0.25	0.14	0.05	1.16	0.19	0.00
20k,30Hz	0.19	0.13	0.05	1.42	0.19	0.00
25k,20Hz	0.26	0.11	0.07	1.13	0.19	0.01
25k,30Hz	0.18	0.13	0.07	1.27	0.20	0.02

Table 14 - Application of General Scaling Law, Eq. (17): Model F

Model F -- Predicted Slopes for Model SFS

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	94.8831	94.3169	92.9175	90.4048	88.9655	80.7807
15k,30Hz	184.2151	180.8103	176.8124	171.8294	168.6868	158.7090
20k,20Hz	91.2608	90.7293	89.5461	87.1237	85.4688	77.9716
20k,30Hz	177.3824	173.0757	169.0246	163.5076	159.6400	152.5777
25k,20Hz	87.7850	87.3189	86.5667	84.0660	82.5218	75.2619
25k,30Hz	170.0811	165.9697	162.2742	156.9833	153.2390	146.8111

Model SFS -- Actual Slopes

15k,20Hz	95.5560	95.0241	93.8909	91.1718	89.5320	81.4344
15k,30Hz	185.4826	182.1611	178.2545	173.2567	170.3437	159.9764
20k,20Hz	91.9091	91.4055	90.4857	87.8731	86.3100	78.5630
20k,30Hz	178.6100	174.3909	170.3892	164.8775	160.9170	153.7541
25k,20Hz	88.4012	87.9814	87.2773	84.7855	83.3375	75.9534
25k,30Hz	171.2538	167.2259	163.6361	158.3070	154.7770	147.9915

% Errors

15k,20Hz	0.70	0.74	1.04	0.84	0.63	0.80
15k,30Hz	0.68	0.74	0.81	0.82	0.97	0.79
20k,20Hz	0.71	0.74	1.04	0.85	0.97	0.75
20k,30Hz	0.69	0.75	0.80	0.83	0.79	0.77
25k,20Hz	0.70	0.75	0.81	0.85	0.98	0.91
25k,30Hz	0.68	0.75	0.83	0.84	0.99	0.80

Model F -- Predicted Slopes for Model LFS

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	42.8205	42.5650	41.9335	40.7995	40.1500	36.4562
15k,30Hz	83.1359	81.5993	79.7951	77.5463	76.1280	71.6250
20k,20Hz	41.1858	40.9460	40.4120	39.3188	38.5719	35.1884
20k,30Hz	80.0523	78.1087	76.2804	73.7906	72.0452	68.8580
25k,20Hz	39.6172	39.4069	39.0674	37.9388	37.2419	33.9655
25k,30Hz	76.7573	74.9018	73.2340	70.8462	69.1564	66.2556

Model LFS -- Actual Slopes

15k,20Hz	43.1493	42.9191	42.4239	41.1542	40.4546	36.7769
15k,30Hz	83.7596	82.2760	80.5439	78.2035	76.9664	72.2526
20k,20Hz	41.5029	41.2848	40.8854	39.6653	38.9983	35.4802
20k,30Hz	80.6564	78.7700	76.9917	74.4286	72.7116	69.4417
25k,20Hz	39.9178	39.7376	39.4354	38.2714	37.6550	34.3016
25k,30Hz	77.3329	75.5321	73.9393	71.4616	69.9363	66.8381

% Errors

15k,20Hz	0.76	0.83	1.16	0.86	0.75	0.87
15k,30Hz	0.74	0.82	0.93	0.84	1.09	0.87
20k,20Hz	0.76	0.82	1.16	0.87	1.09	0.82
20k,30Hz	0.75	0.84	0.92	0.86	0.92	0.84
25k,20Hz	0.75	0.83	0.93	0.87	1.10	0.98
25k,30Hz	0.74	0.83	0.95	0.86	1.12	0.87

Table 15 - Application of Scaling Rule, Eq. (20): Model B

Model B -- Predicted Slopes for Model SB

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	44.9686	43.6083	42.8322	41.4908	40.1432	36.2458
15k,30Hz	84.7867	83.4323	81.6506	79.3615	78.0137	72.3979
20k,20Hz	42.9501	41.5778	40.9606	39.7807	38.5474	34.7454
20k,30Hz	80.4640	79.1624	77.4597	75.3642	74.1406	68.9877
25k,20Hz	41.0202	39.8811	39.4133	38.1492	37.1925	33.8858
25k,30Hz	76.2733	75.1062	73.6088	71.6275	70.5716	65.6862

Model SB -- Actual Slopes

15k,20Hz	47.4564	44.1972	42.8883	41.6649	40.4632	40.4322
15k,30Hz	89.4842	83.4618	82.5339	82.4570	80.1964	71.0861
20k,20Hz	44.9945	41.8477	40.8071	39.8977	38.8161	37.3083
20k,30Hz	83.7895	78.3176	77.4541	77.5448	75.4095	66.6908
25k,20Hz	42.7108	39.7092	39.1862	38.7502	37.7129	34.6656
25k,30Hz	78.5396	73.5559	72.7762	72.9090	70.9778	62.6799

% Errors

15k,20Hz	5.24	1.33	0.13	0.42	0.79	10.35
15k,30Hz	5.25	0.04	1.07	3.75	2.72	1.85
20k,20Hz	4.54	0.65	0.38	0.29	0.69	6.87
20k,30Hz	3.97	1.08	0.01	2.81	1.68	3.44
25k,20Hz	3.96	0.43	0.58	1.55	1.38	2.25
25k,30Hz	2.89	2.11	1.14	1.76	0.57	4.80

Model B -- Predicted Slopes for Model LB

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	68.4414	66.3710	65.1898	63.1483	61.0973	55.1654
15k,30Hz	129.0438	126.9825	124.2707	120.7867	118.7354	110.1883
20k,20Hz	65.3693	63.2806	62.3413	60.5456	58.6684	52.8818
20k,30Hz	122.4647	120.4837	117.8922	114.7029	112.8406	104.9980
25k,20Hz	62.4320	60.6984	59.9863	58.0624	56.6063	51.5736
25k,30Hz	116.0865	114.3102	112.0312	109.0157	107.4087	99.9732

Model LB -- Actual Slopes

15k,20Hz	65.7241	63.2517	62.1980	62.0840	60.5798	54.3977
15k,30Hz	124.5848	120.6344	119.2762	118.5159	116.5450	109.8431
20k,20Hz	63.2717	60.8687	60.1370	59.6891	58.3248	52.8371
20k,30Hz	118.6726	115.3461	114.3391	113.7523	111.5107	105.4689
25k,20Hz	60.9737	58.8222	57.8487	57.6041	56.4100	51.2740
25k,30Hz	113.4698	110.4936	109.4355	109.0340	106.8834	101.3200

% Errors

15k,20Hz	4.13	4.93	4.81	1.71	0.85	1.41
15k,30Hz	3.58	5.26	4.19	1.92	1.88	0.31
20k,20Hz	3.32	3.96	3.67	1.43	0.59	0.08
20k,30Hz	3.20	4.45	3.11	0.84	1.19	0.45
25k,20Hz	2.39	3.19	3.70	0.80	0.35	0.58
25k,30Hz	2.31	3.45	2.37	0.02	0.49	1.33

Table 16 - Application of Scaling
Rule, Eq. (20): Model F

Model F -- Predicted Slopes for Model SF

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	49.2863	48.9922	48.2653	46.9601	46.2125	41.9609
15k,30Hz	95.6891	93.9205	91.8438	89.2555	87.6231	82.4402
20k,20Hz	47.4048	47.1287	46.5141	45.2558	44.3961	40.5018
20k,30Hz	92.1399	89.9029	87.7985	84.9328	82.9238	79.2553
25k,20Hz	45.5993	45.3572	44.9664	43.6674	42.8653	39.0942
25k,30Hz	88.3474	86.2117	84.2921	81.5438	79.5988	76.2599

Model SF -- Actual Slopes

15k,20Hz	53.3841	51.5583	50.1987	48.2431	47.7424	41.5189
15k,30Hz	100.0835	96.3716	94.8199	91.5796	92.5901	81.1789
20k,20Hz	50.8026	49.1736	47.8806	46.0006	45.5712	39.6418
20k,30Hz	94.8140	91.3715	89.5969	86.7631	88.0611	77.1814
25k,20Hz	48.4111	47.0004	45.7166	44.0171	43.6058	38.2396
25k,30Hz	90.0473	86.7471	85.1779	82.4775	83.7859	73.3774

% Errors

15k,20Hz	7.68	4.98	3.85	2.66	3.20	1.06
15k,30Hz	4.39	2.54	3.14	2.54	5.36	1.55
20k,20Hz	6.69	4.16	2.85	1.62	2.58	2.17
20k,30Hz	2.82	1.61	2.01	2.11	5.83	2.69
25k,20Hz	5.81	3.50	1.64	0.79	1.70	2.23
25k,30Hz	1.89	0.62	1.04	1.13	5.00	3.93

Model F -- Predicted Slopes for Model LF

	600#	900#	1160#	1450#	1740#	3625#
15k,20Hz	82.6656	82.1724	80.9531	78.7640	77.5100	70.3791
15k,30Hz	160.4950	157.5286	154.0454	149.7041	146.9662	138.2731
20k,20Hz	79.5098	79.0467	78.0159	75.9054	74.4636	67.9318
20k,30Hz	154.5420	150.7899	147.2604	142.4538	139.0842	132.9313
25k,20Hz	76.4815	76.0755	75.4201	73.2414	71.8960	65.5709
25k,30Hz	148.1810	144.5989	141.3792	136.7696	133.5074	127.9073

Model LF -- Actual Slopes

15k,20Hz	75.4718	75.1549	72.1943	73.6001	72.0289	67.0830
15k,30Hz	148.7206	144.3879	141.1981	139.4032	138.2484	129.5165
20k,20Hz	73.2373	73.0159	70.0840	71.5287	69.9180	65.2167
20k,30Hz	143.6918	139.0496	136.6954	134.8502	133.8238	125.0736
25k,20Hz	71.2348	70.9547	68.0336	69.5293	67.9277	63.4332
25k,30Hz	138.8957	134.9244	132.9889	131.2271	129.6827	121.3504

% Errors

15k,20Hz	9.53	9.34	12.13	7.02	7.61	4.91
15k,30Hz	7.92	9.10	9.10	7.39	6.31	6.76
20k,20Hz	8.56	8.26	11.32	6.12	6.50	4.16
20k,30Hz	7.55	8.44	7.73	5.64	3.93	6.28
25k,20Hz	7.37	7.22	10.86	5.34	5.84	3.37
25k,30Hz	6.69	7.17	6.31	4.22	2.95	5.40

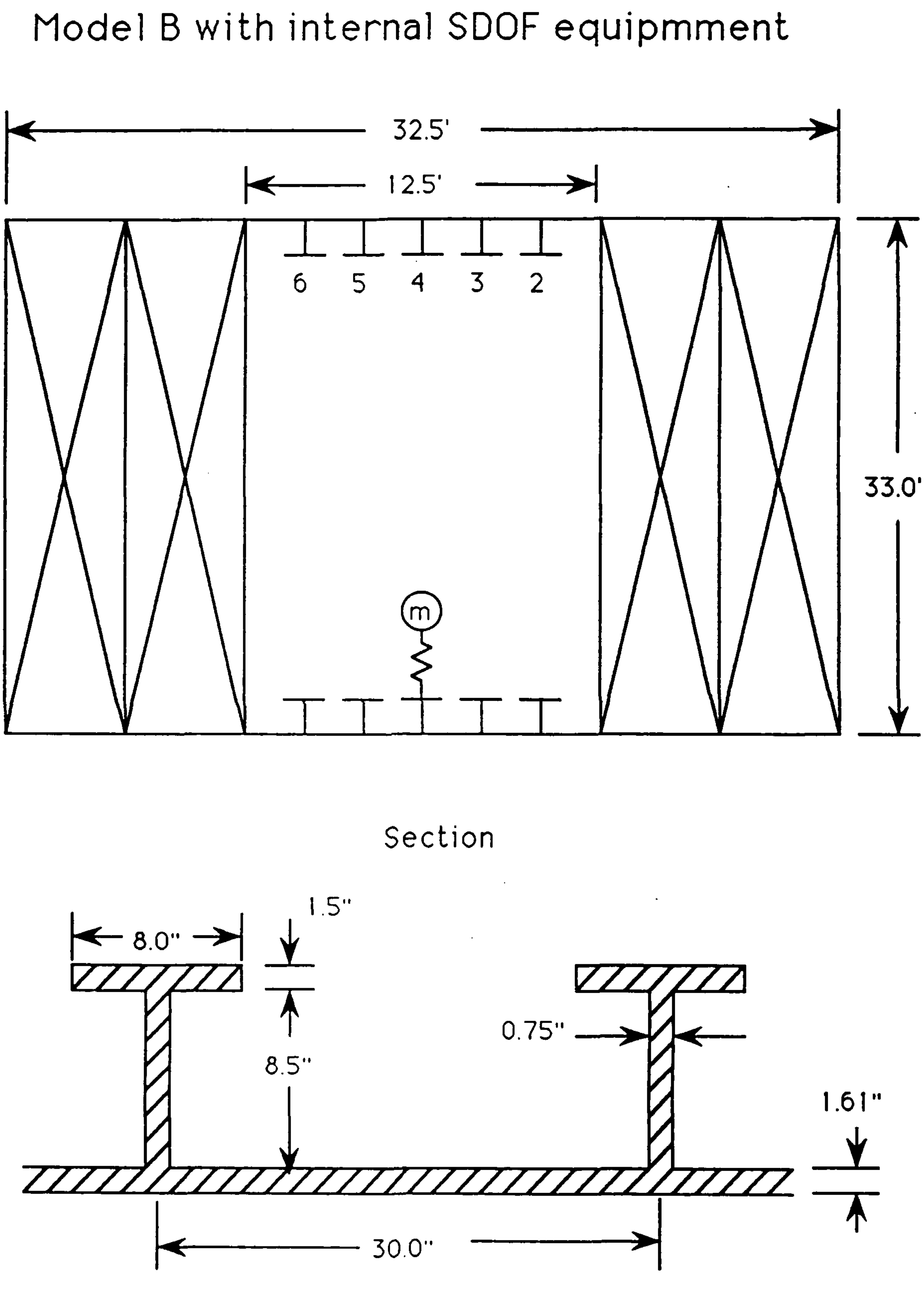
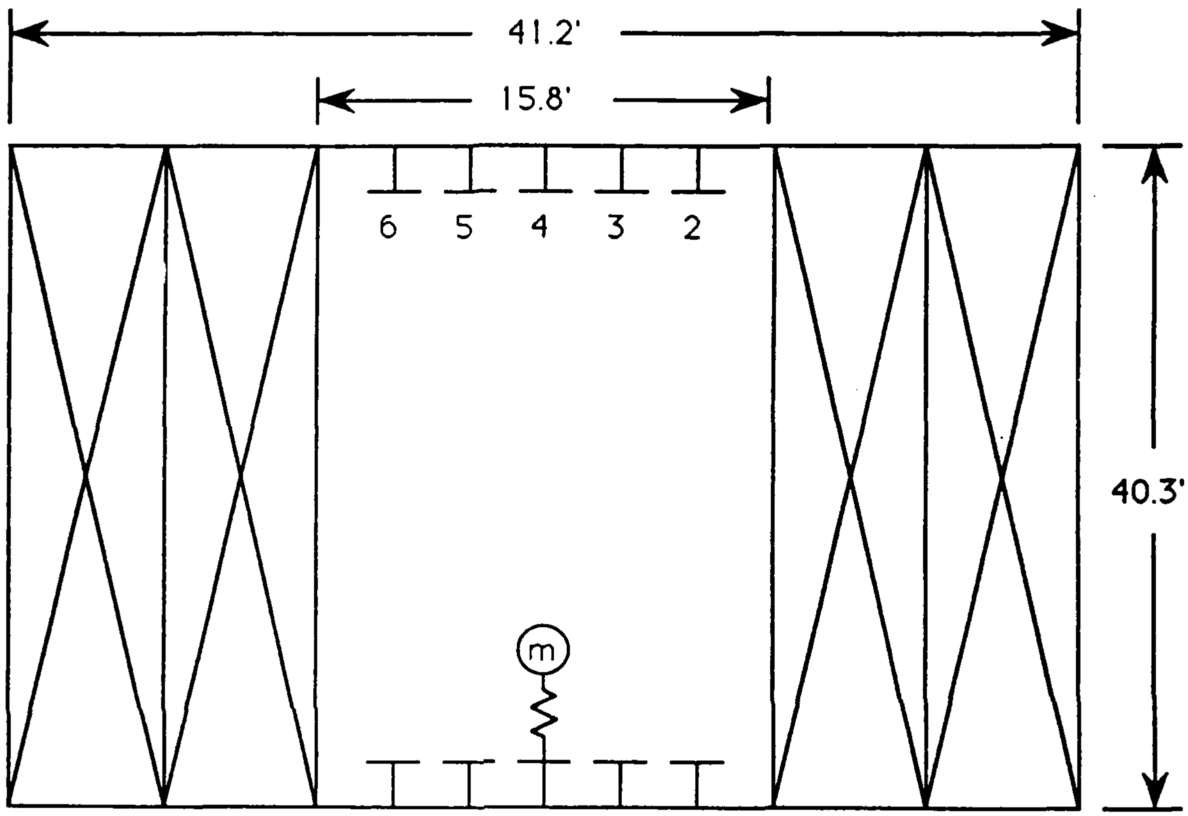

Model B with internal SDOF equipmment
32.5'
12.5'
6
5
4
3
2
33.0'
m
Section
8.0"
1.5"
8.5"
0.75"
1.61"
30.0"

Figure 1A

Model F with internal SDOF equipmment



Section

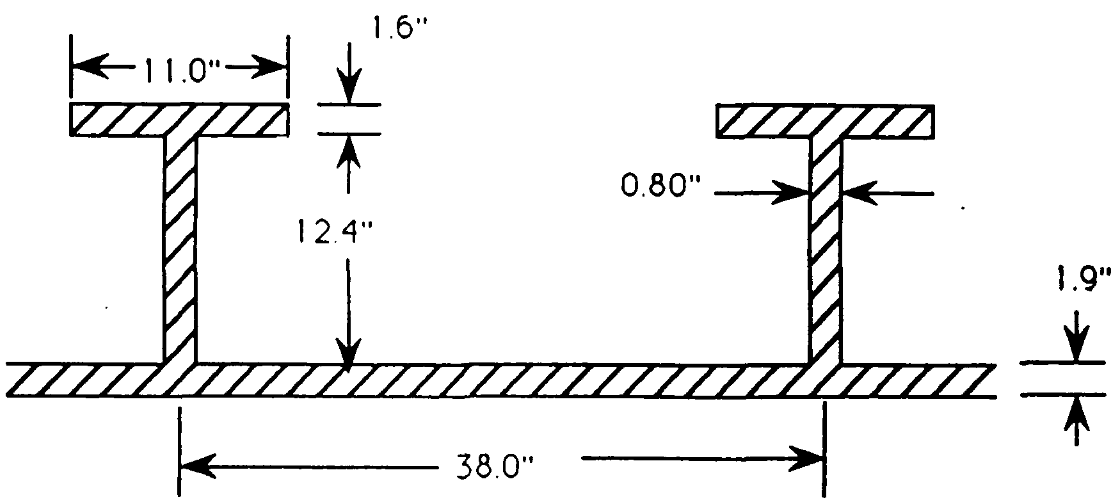


Figure 1B

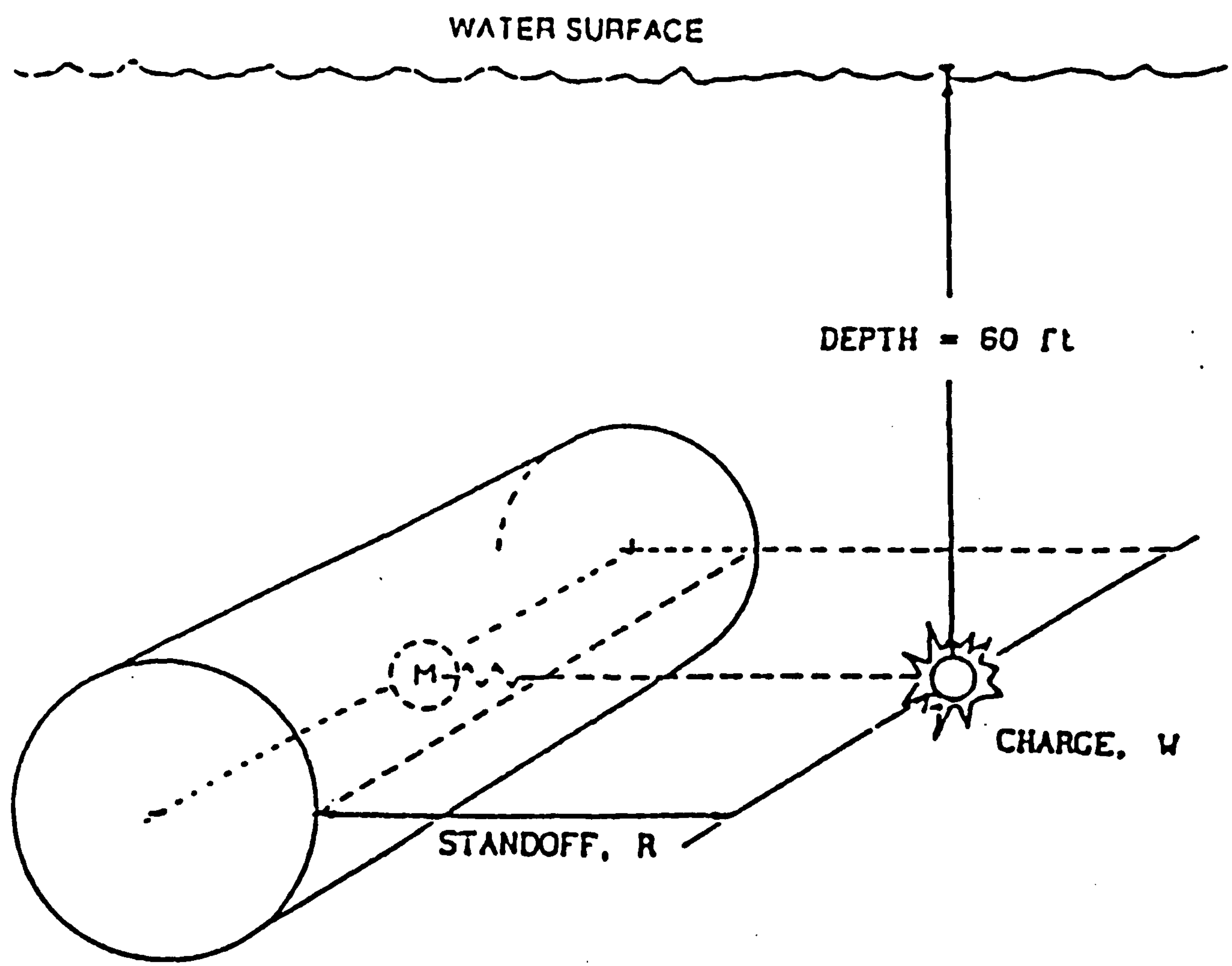
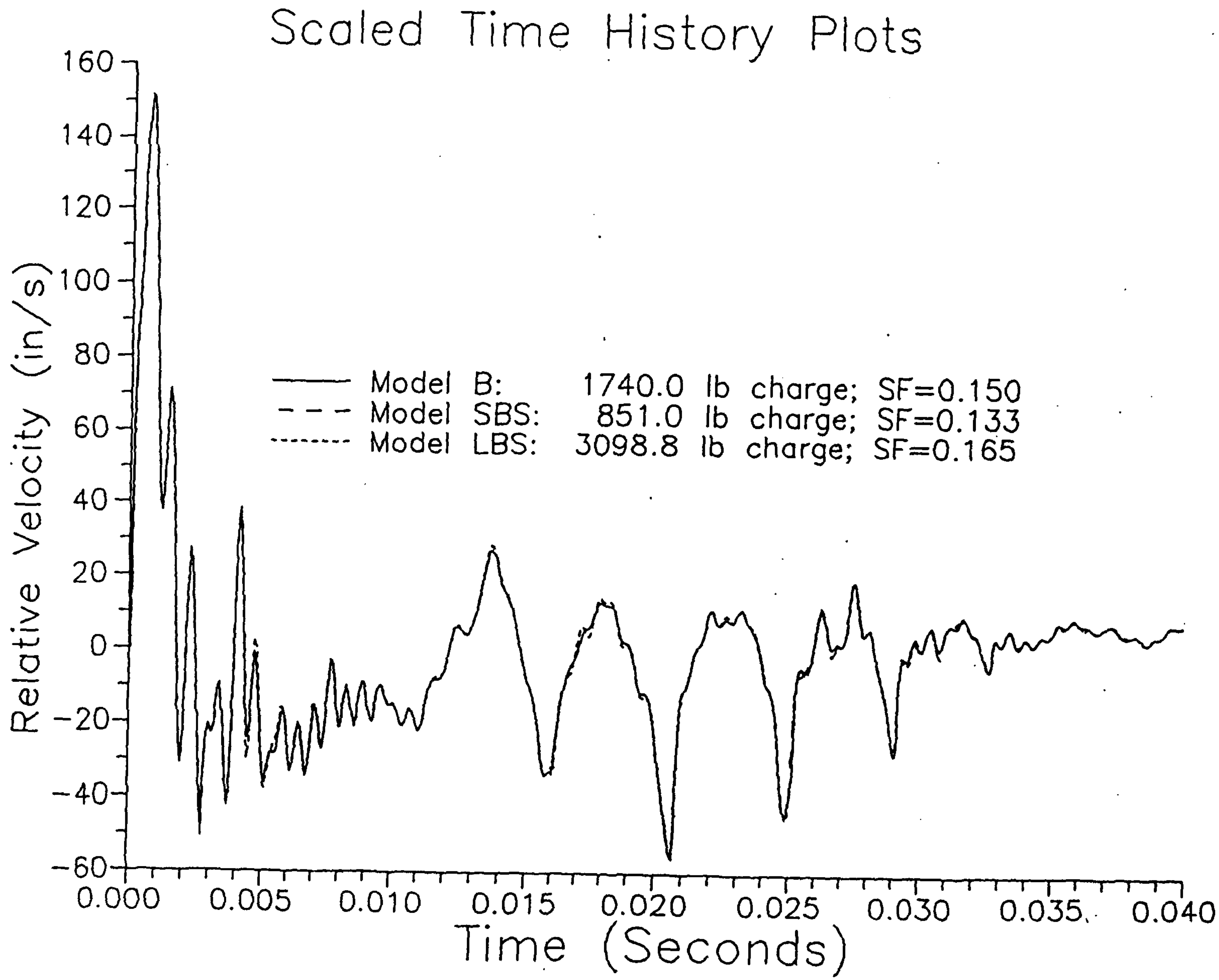


Figure 2



Scaled Time History Plots
Model B: 1740.0 lb charge; SF=0.150
Model SBS: 851.0 lb charge; SF=0.133
Model LBS: 3098.8 lb charge; SF=0.165
Relative Velocity (in/s)
160
140
120
100
80
60
40
20
0
−20
−40
−60
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
Time (Seconds)

Figure 3

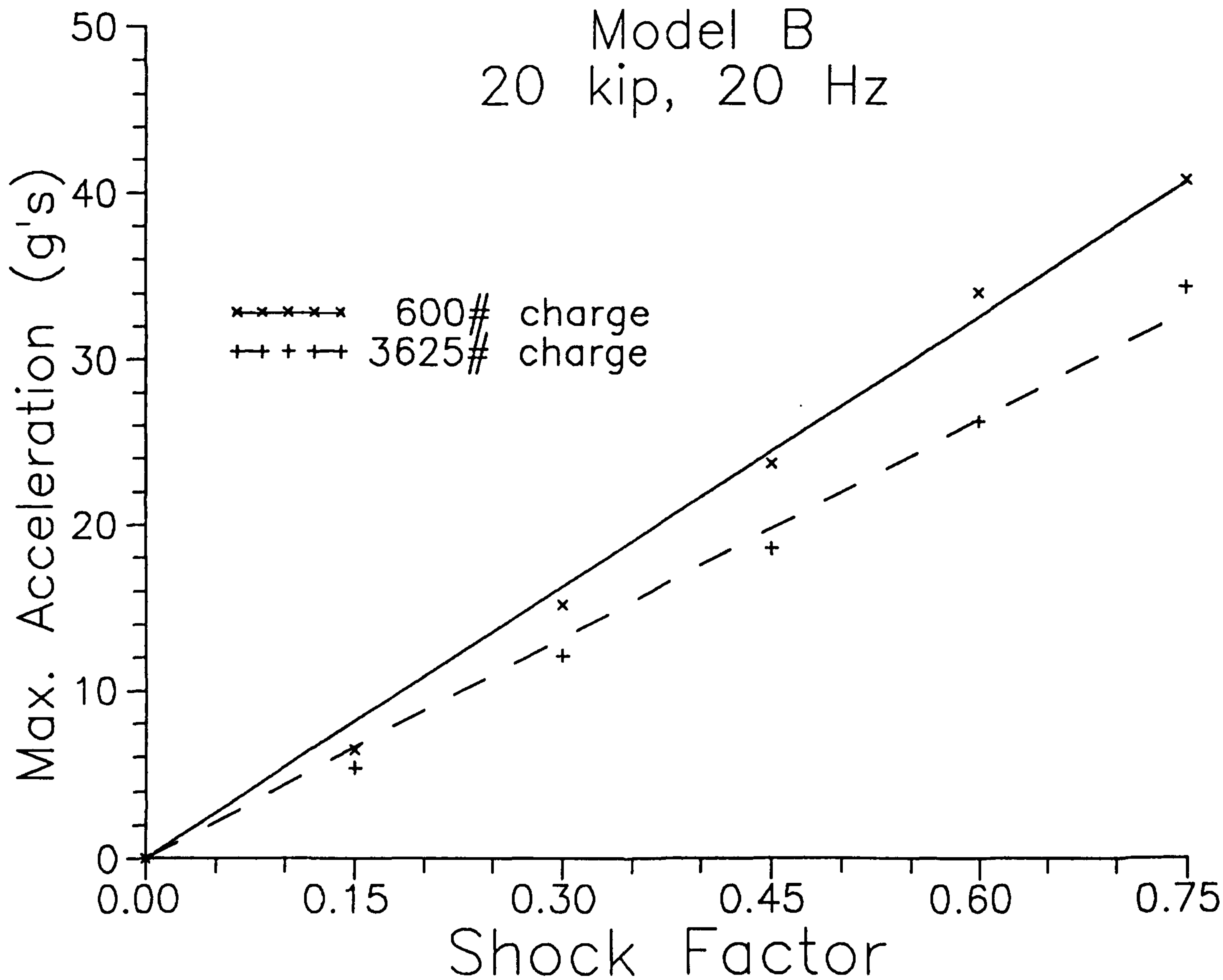

Model B
20 kip, 20 Hz
600# charge
3625# charge
Max. Acceleration (g's)
50
40
30
20
10
0
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor

Figure 4

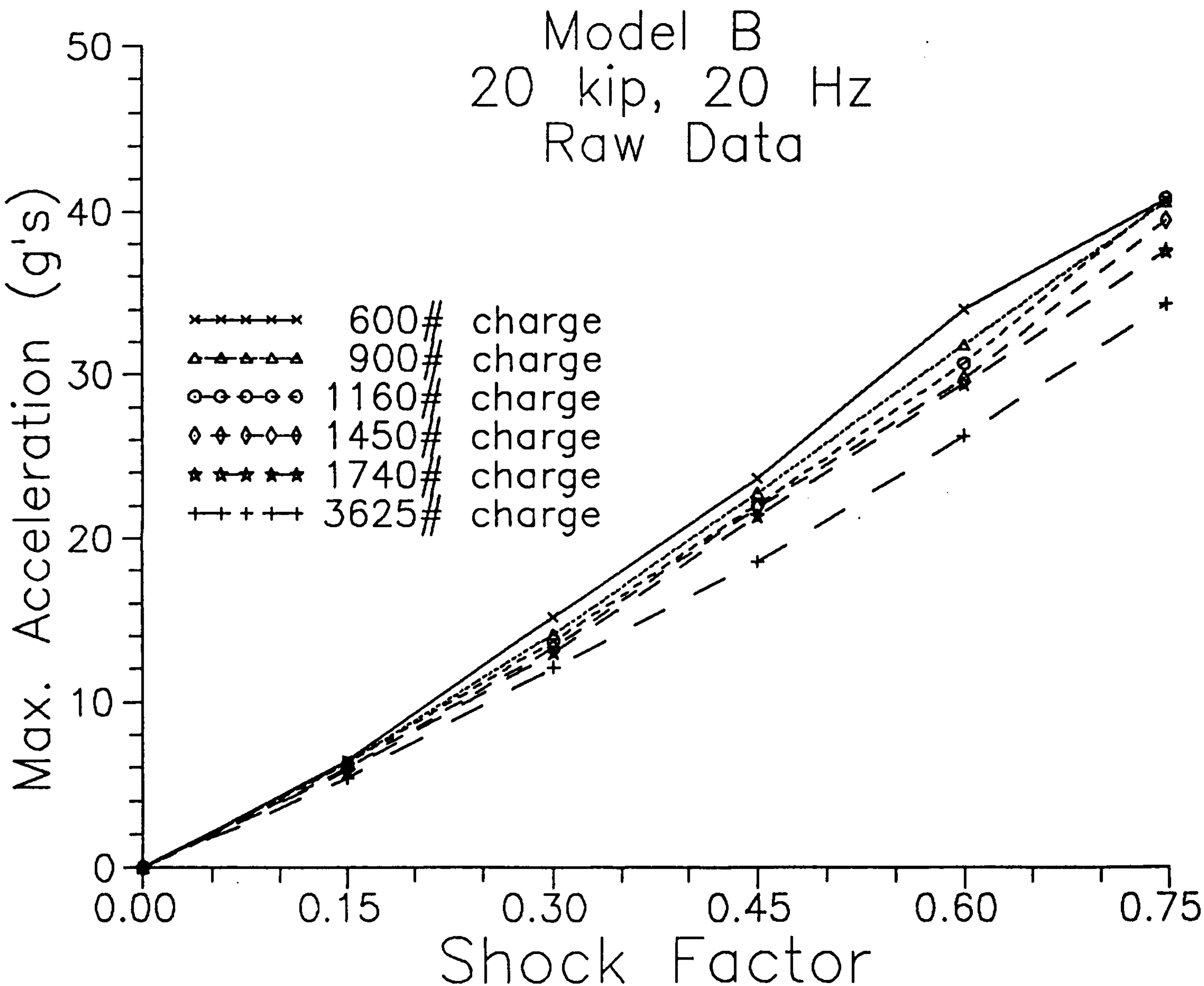
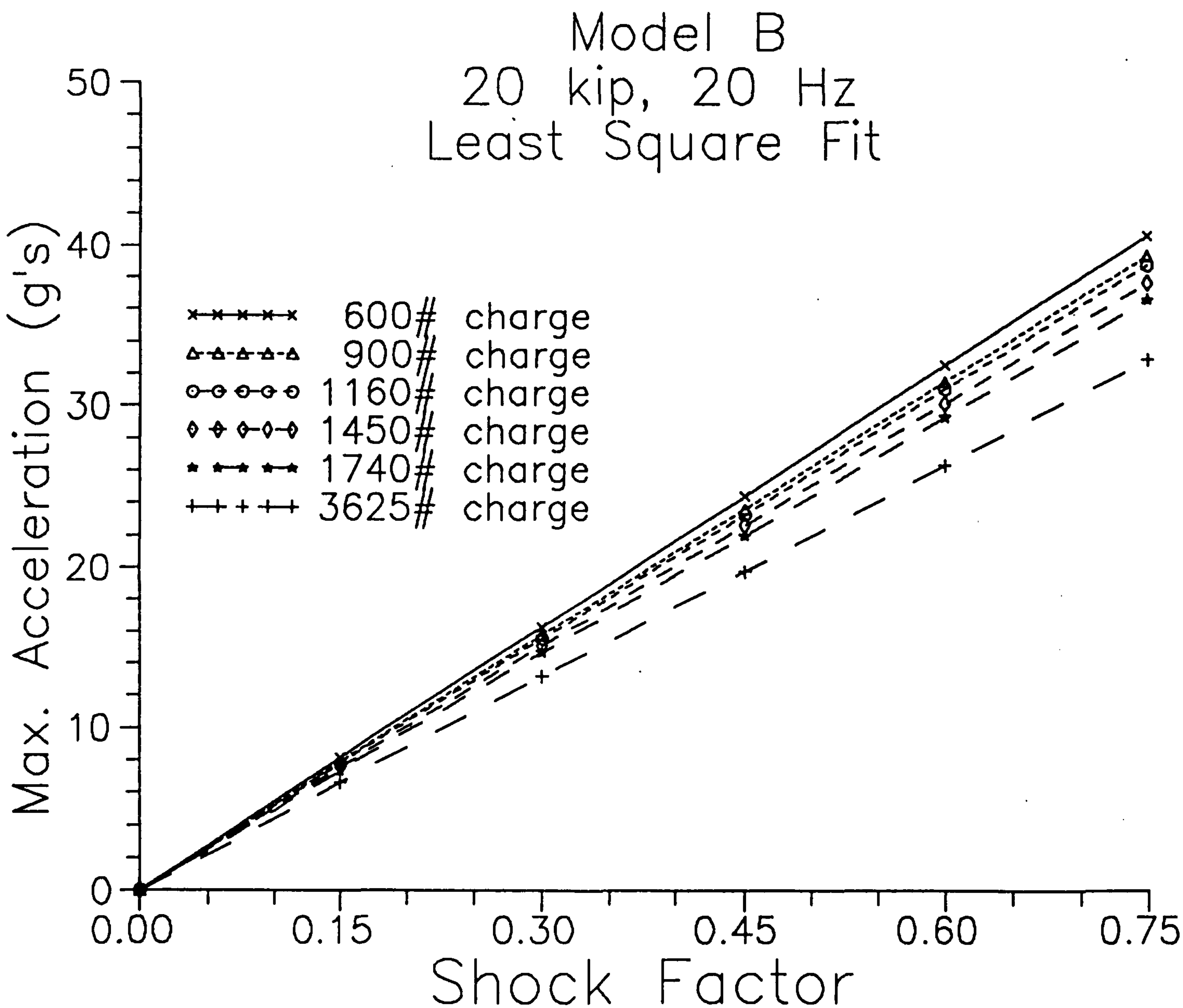
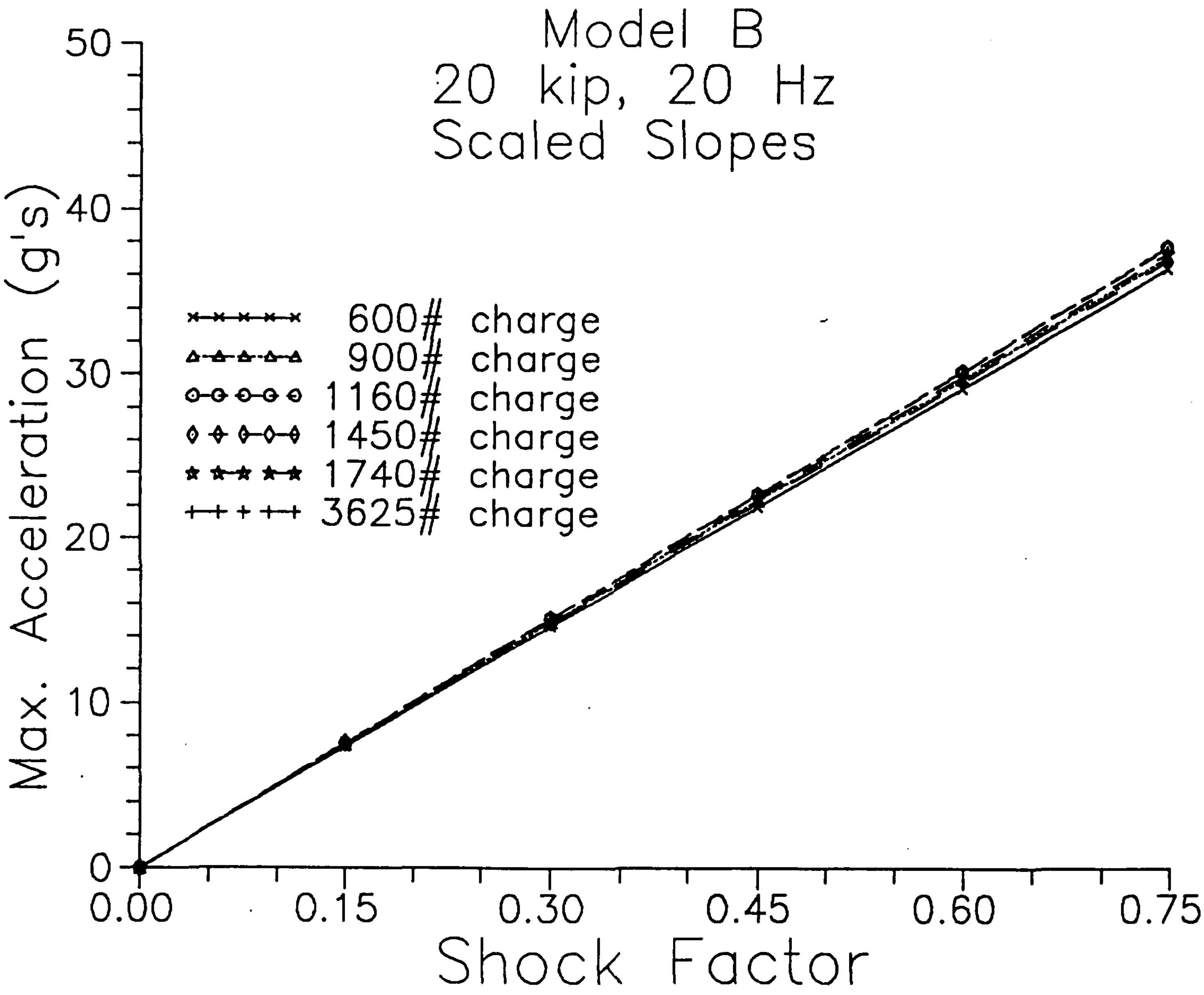

Model B
20 kip, 20 Hz
Raw Data
600# charge
900# charge
1160# charge
1450# charge
1740# charge
3625# charge
Max. Acceleration (g's)
0
10
20
30
40
50
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor

Figure 5



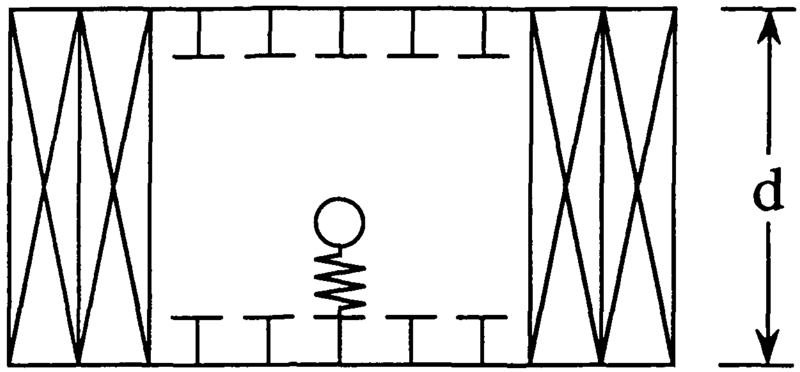
Model B
20 kip, 20 Hz
Least Square Fit
600# charge
900# charge
1160# charge
1450# charge
1740# charge
3625# charge
Max. Acceleration (g's)
Shock Factor
0
10
20
30
40
50
0.00
0.15
0.30
0.45
0.60
0.75

Figure 6

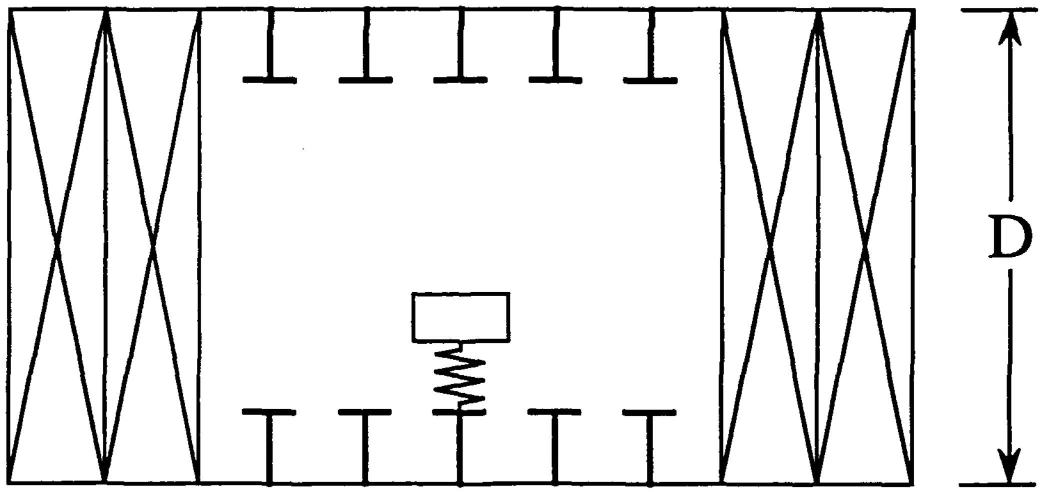


Model B
20 kip, 20 Hz
Scaled Slopes
600# charge
900# charge
1160# charge
1450# charge
1740# charge
3625# charge
Max. Acceleration (g's)
50
40
30
20
10
0
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor

Figure 7

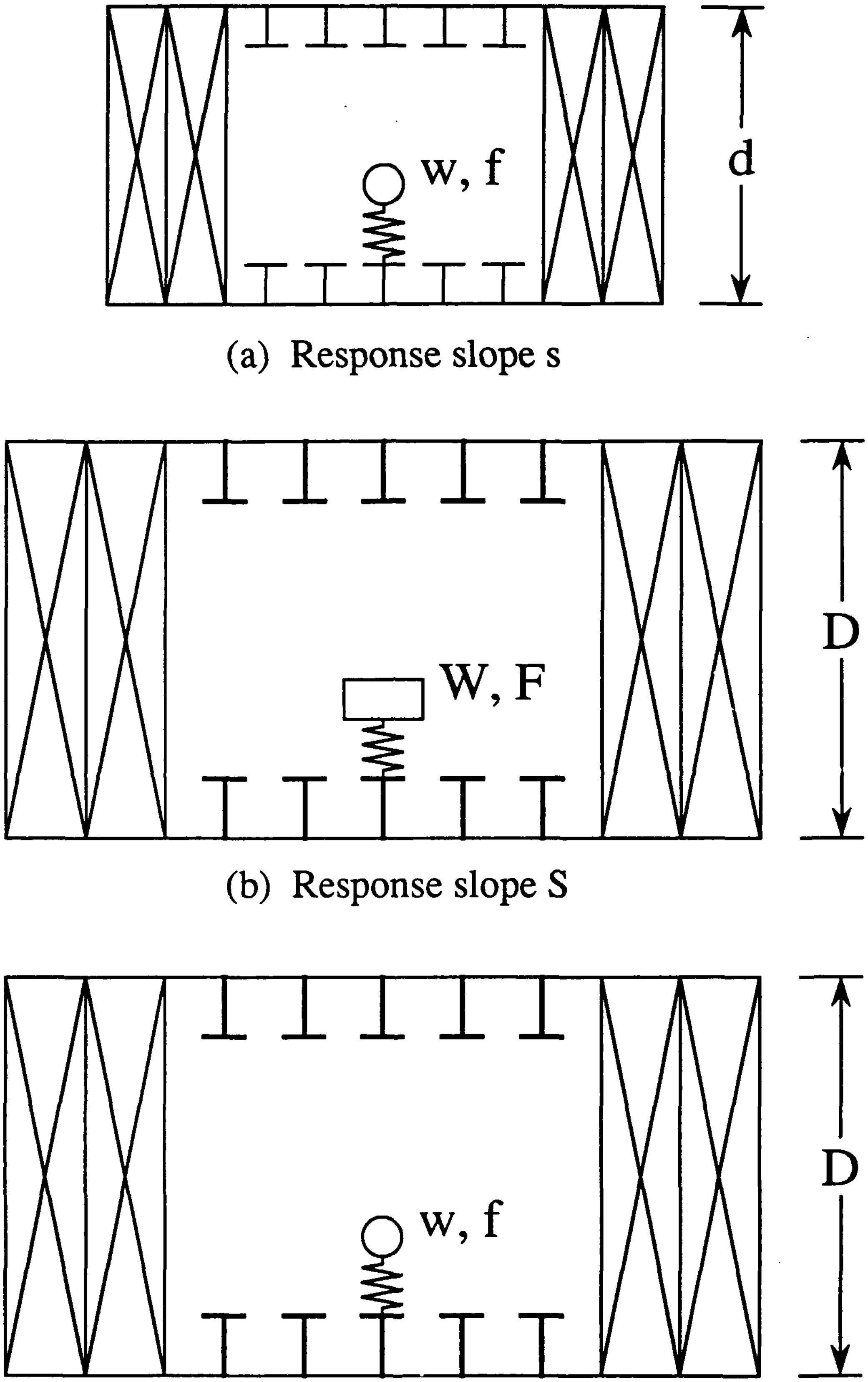


(a)



(b)

Figure 8



(a) Response slope s

(b) Response slope S

(c) Response slope S'

Figure 9

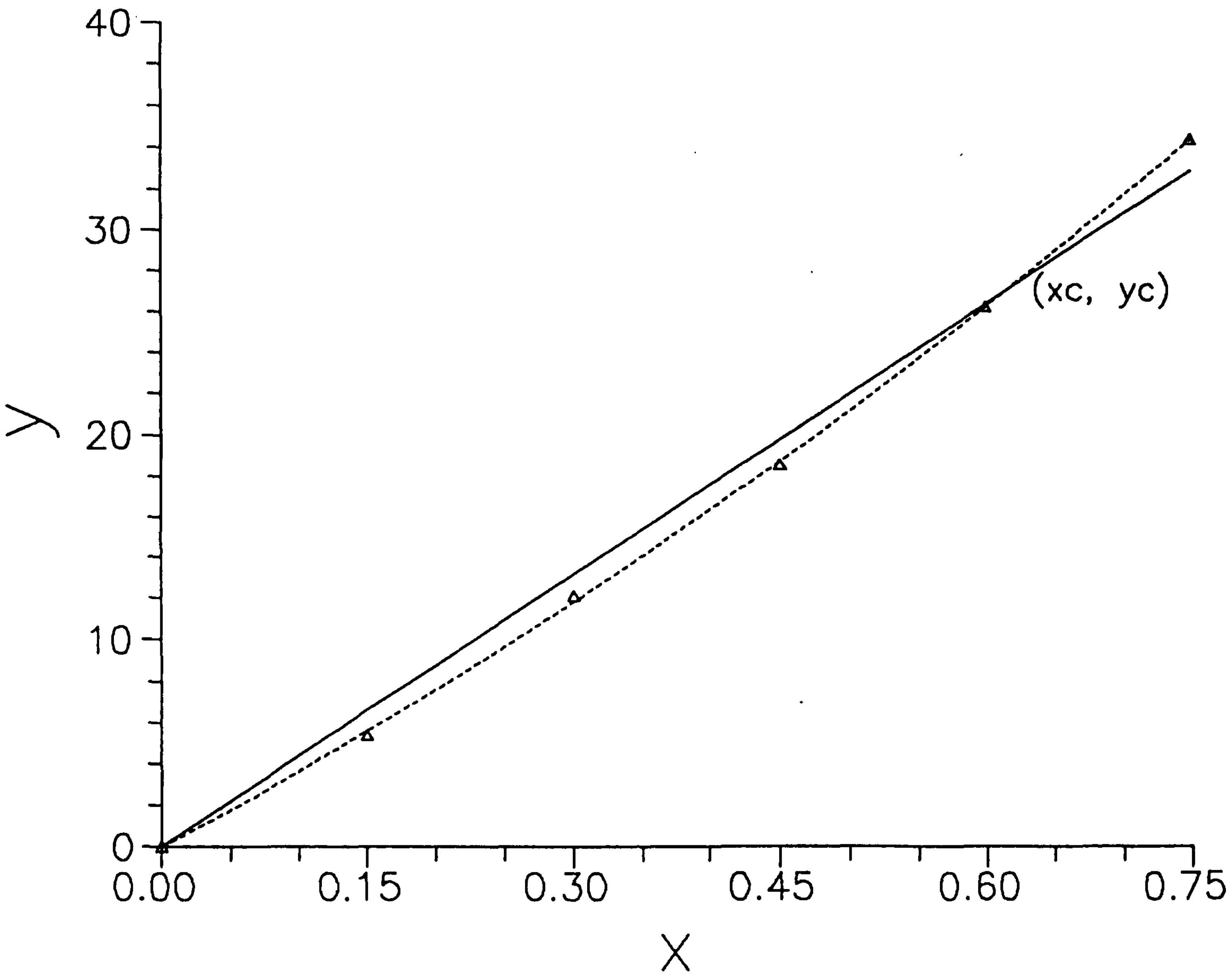


Figure 10

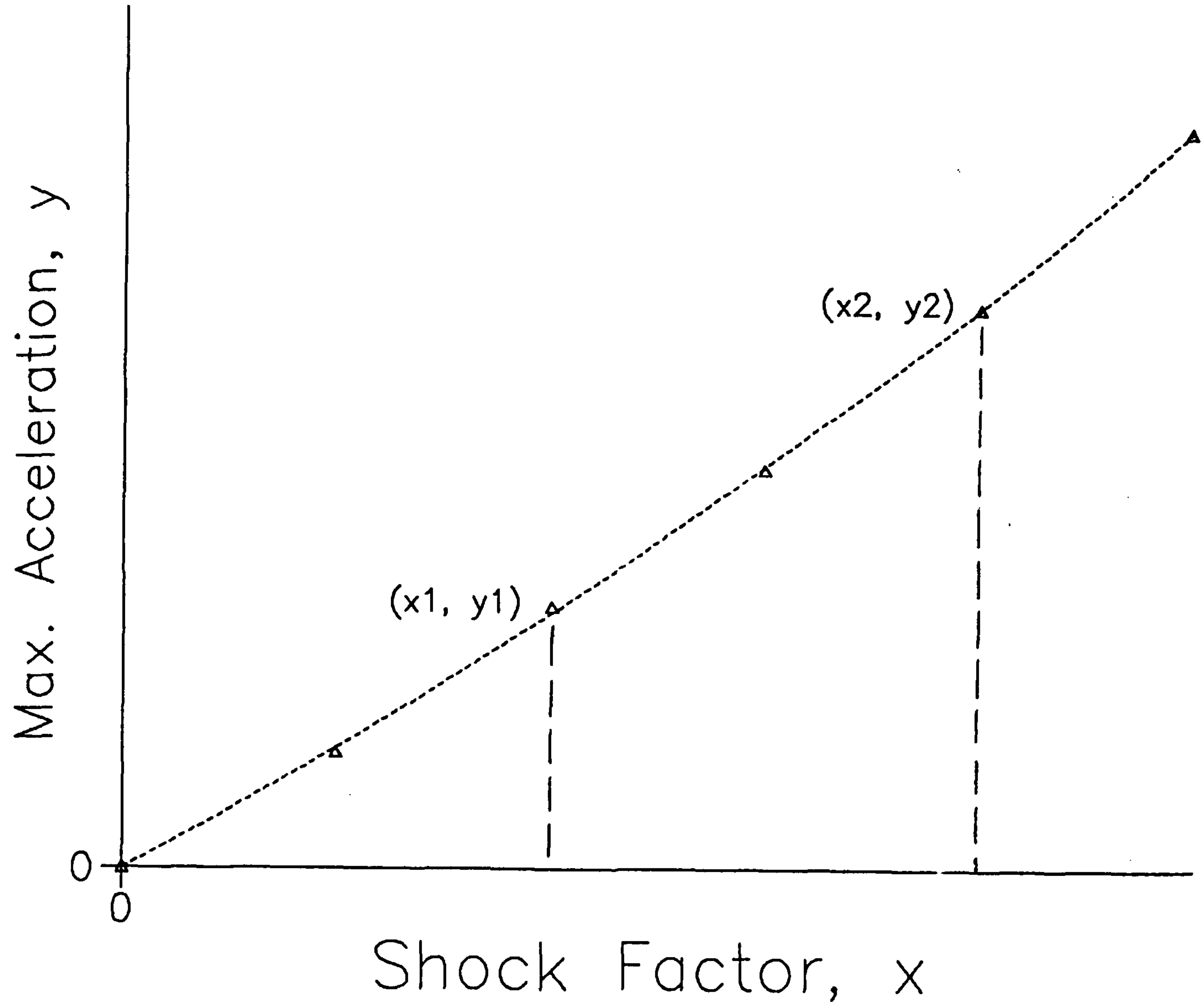


Figure 11

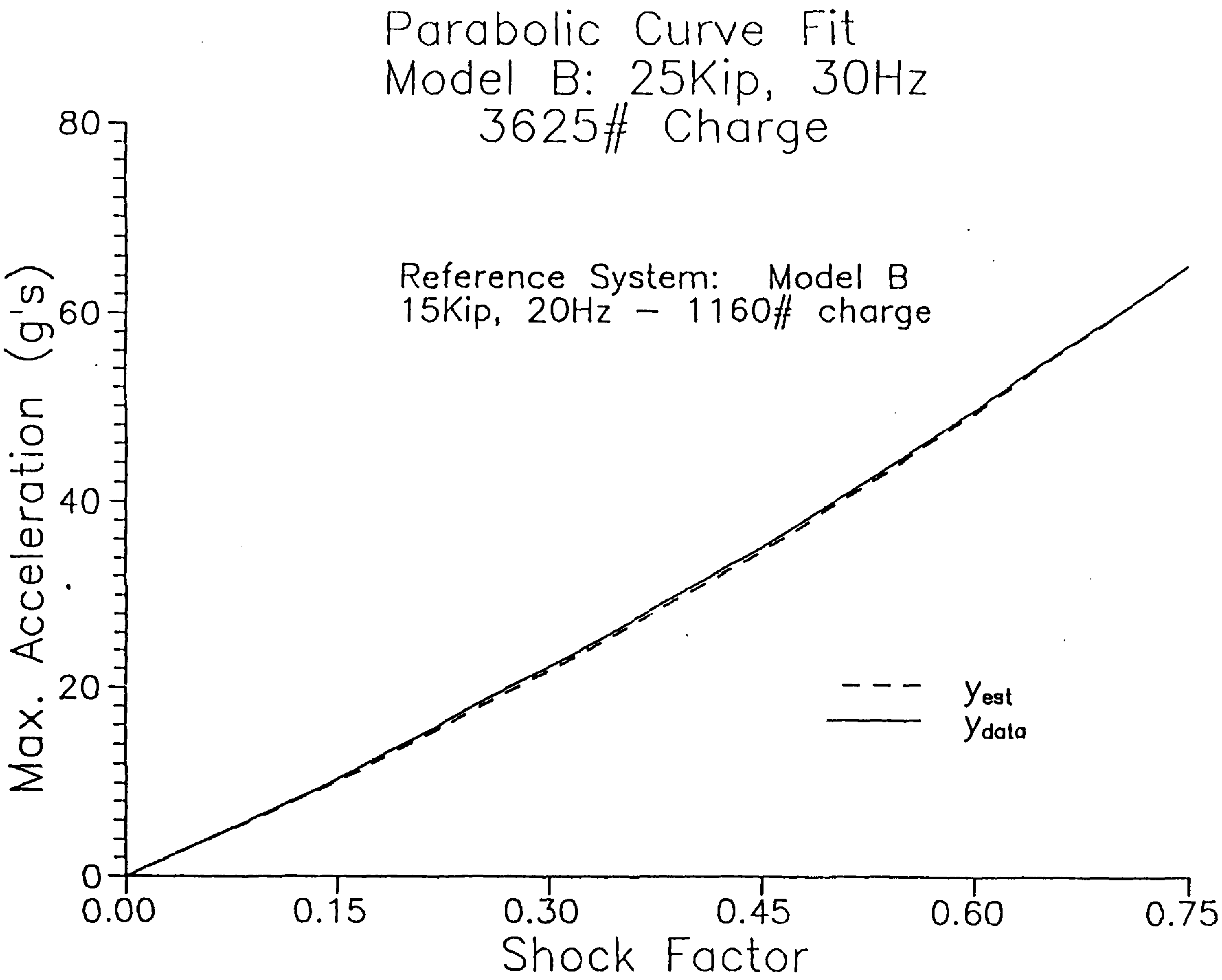

Parabolic Curve Fit
Model B: 25Kip, 30Hz
3625# Charge
Reference System: Model B
15Kip, 20Hz – 1160# charge
y_est
y_data
Max. Acceleration (g's)
Shock Factor
0
20
40
60
80
0.00
0.15
0.30
0.45
0.60
0.75

Figure 12

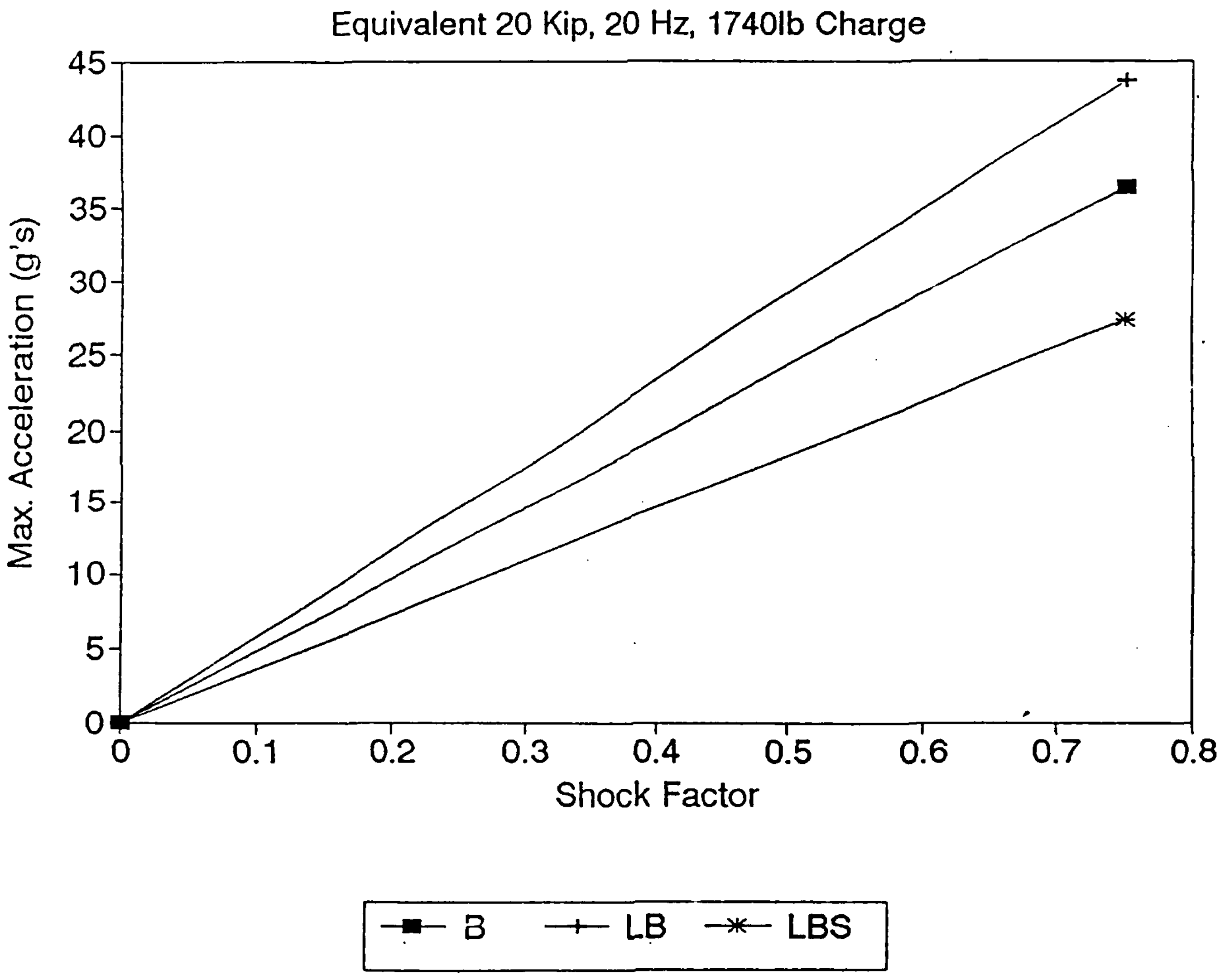
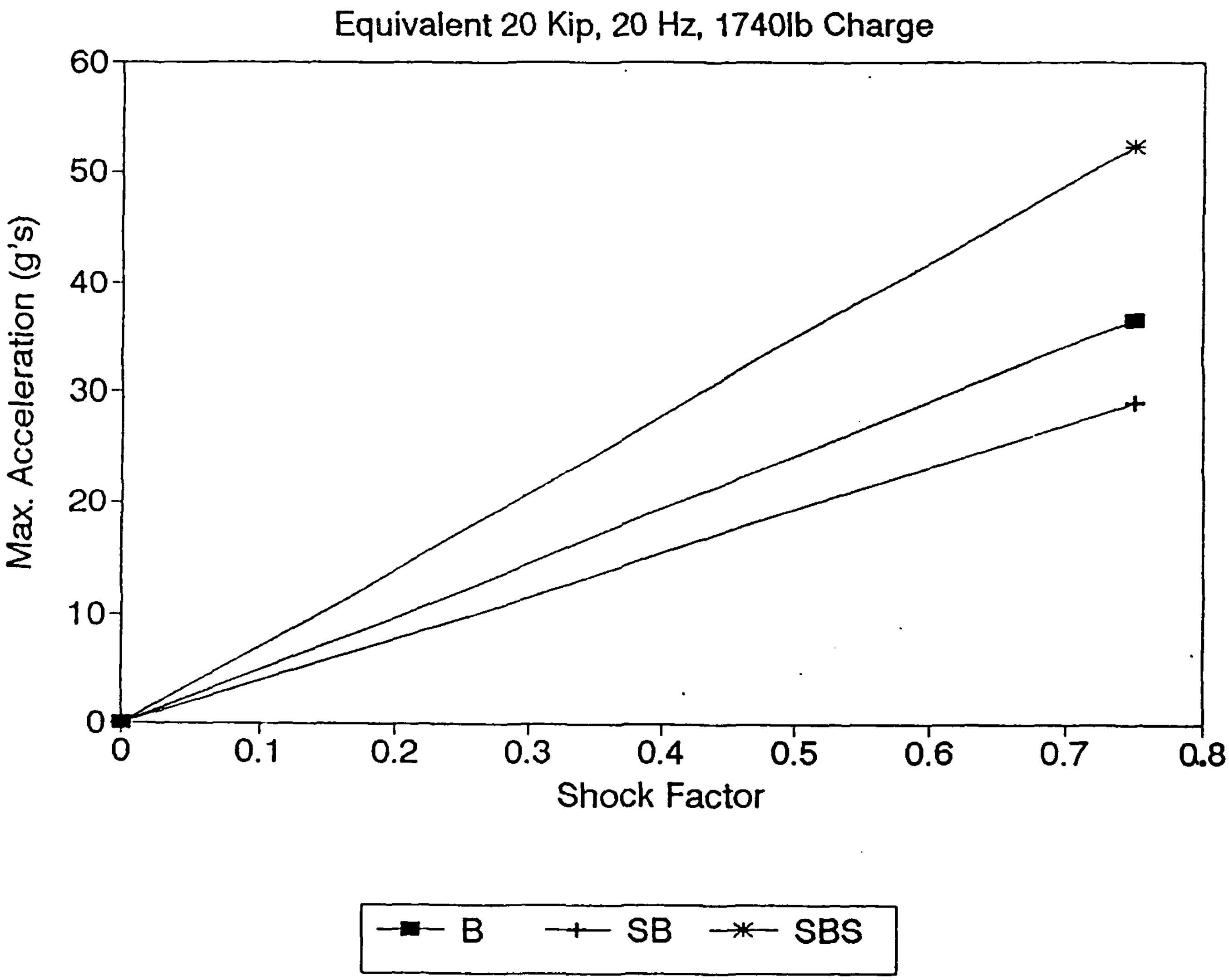

Equivalent 20 Kip, 20 Hz, 1740lb Charge
Max. Acceleration (g's)
45
40
35
30
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Shock Factor
B
LB
LBS

Figure 13



Equivalent 20 Kip, 20 Hz, 1740lb Charge
Max. Acceleration (g's)
0
10
20
30
40
50
60
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Shock Factor
B
SB
SBS

Figure 14

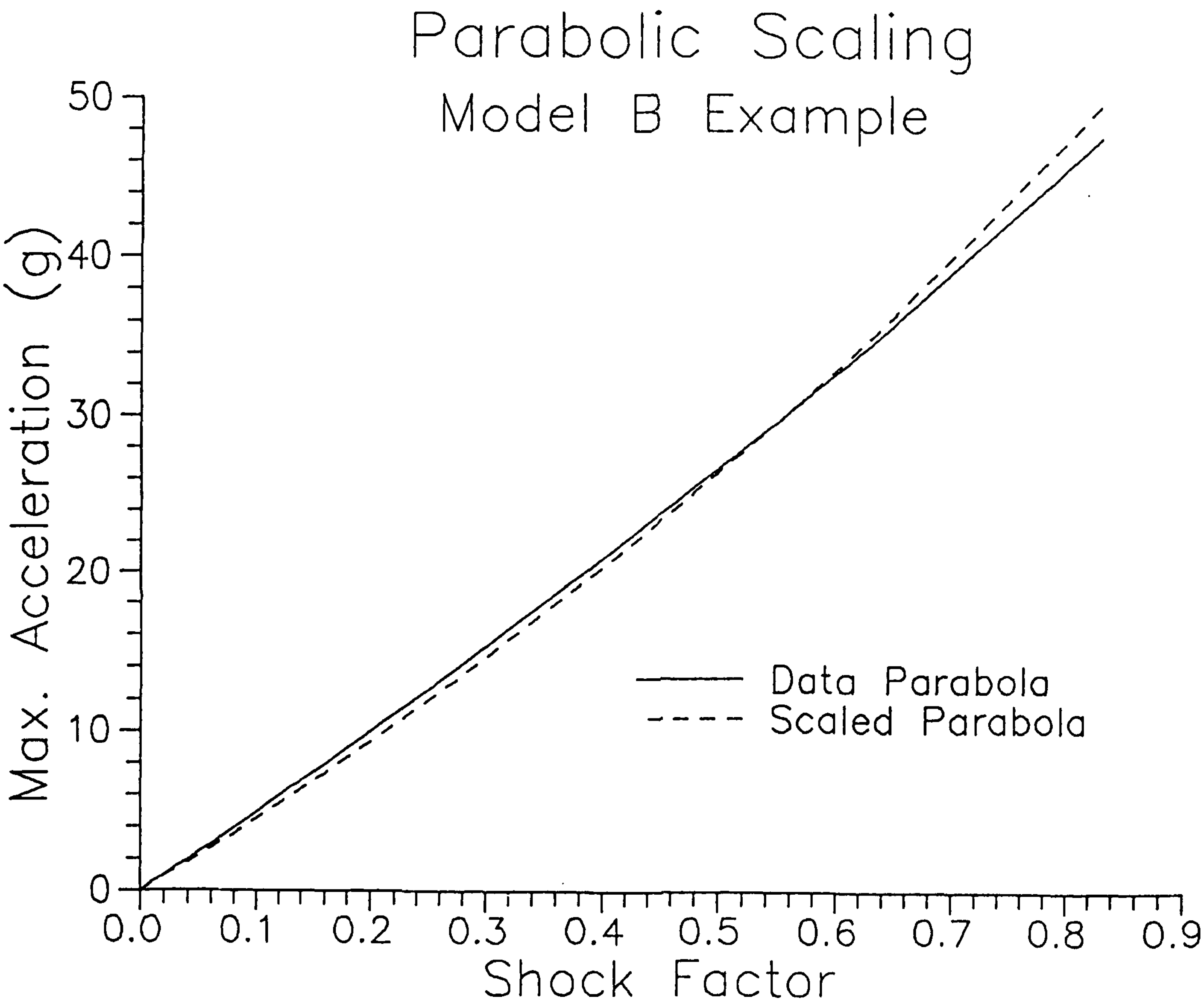
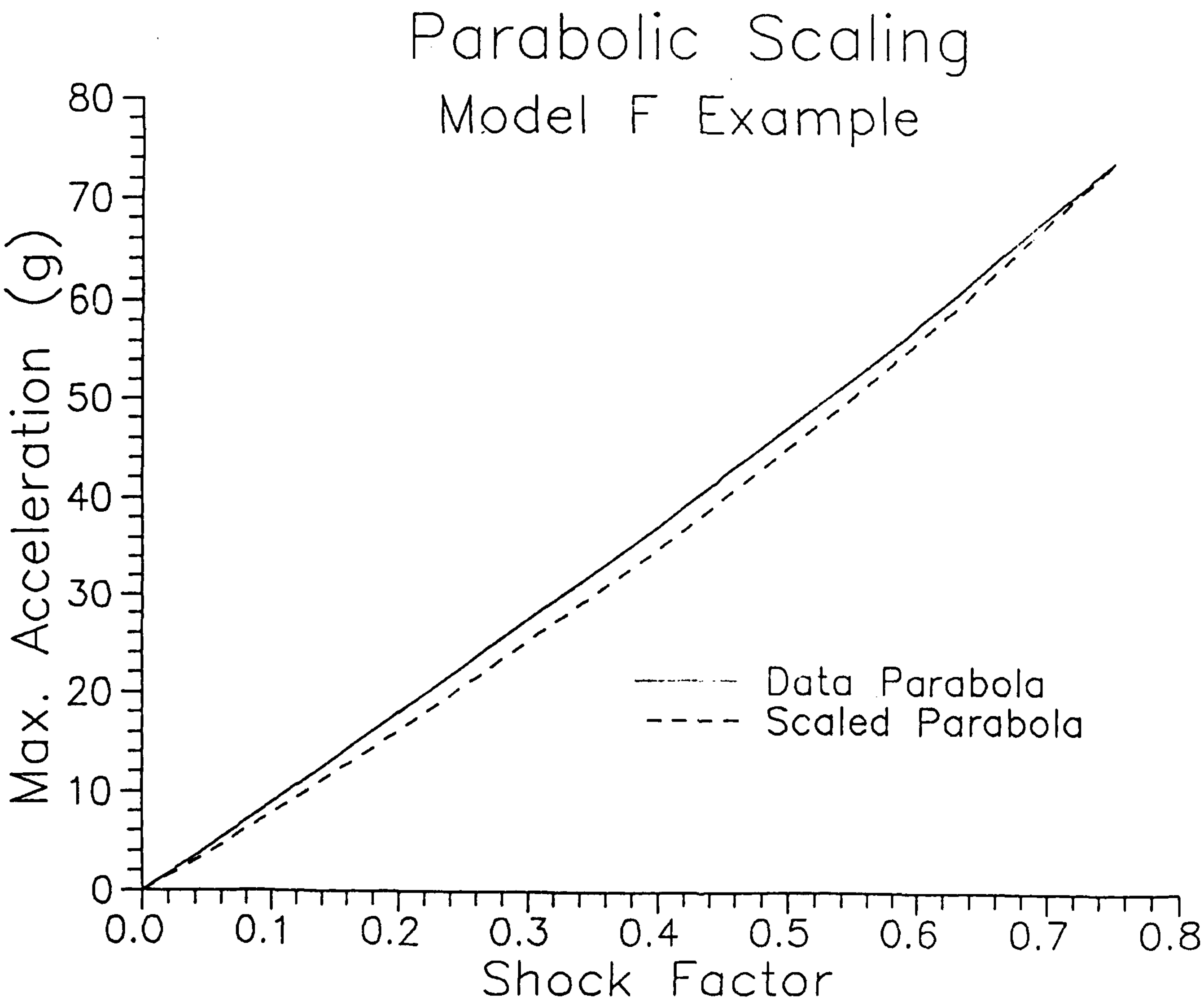

Parabolic Scaling
Model B Example
Max. Acceleration (g)
0
10
20
30
40
50
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Shock Factor
Data Parabola
Scaled Parabola

Figure 15



Parabolic Scaling
Model F Example
Max. Acceleration (g)
80
70
60
50
40
30
20
10
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Shock Factor
Data Parabola
Scaled Parabola

Figure 16

APPENDIX A - Summary of data for Model B

Table A1: Model B

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.3956	6.5686	6.5073	6.2041	6.0219	5.4830
0.30	15.5715	14.5199	13.9880	13.5847	13.2670	12.6493
0.45	24.8191	23.8275	23.0325	22.3908	22.2268	18.9597
0.60	35.6442	33.3863	32.1668	31.0490	30.3143	27.3157
0.75	42.7005	42.6560	42.7179	41.4267	39.4709	36.0714
slope	56.7364	55.0201	54.0409	52.3485	50.6482	45.7309

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	13.3887	13.04272	12.8666	12.45935	12.1329	10.8451
0.30	31.7052	30.17431	28.6857	27.8618	27.2055	24.4456
0.45	46.7465	48.43918	46.9695	45.67122	44.5929	38.8641
0.60	66.4412	62.73582	60.0099	58.72572	61.0938	54.7966
0.75	79.9471	79.75784	79.7418	77.19367	73.4683	71.6140
slope	106.9744	105.2656	103.0176	100.1294	98.4290	91.3436

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.3971	6.2860	6.2250	5.9436	5.7879	5.3364
0.30	15.1429	14.0986	13.6857	13.2914	12.9770	12.0856
0.45	23.6159	22.7440	22.0331	21.8526	21.3429	18.5255
0.60	33.9585	31.7868	30.6274	29.6580	29.3813	26.1935
0.75	40.7400	40.5836	40.8301	39.4717	37.5882	34.3608
slope	54.1896	52.4582	51.6795	50.1909	48.6348	43.8378

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	12.7695	12.40776	12.2366	11.84915	11.5385	10.7133
0.30	30.7301	28.71582	27.2771	26.49537	25.8732	23.2793
0.45	44.3716	46.14995	44.7531	43.51862	42.4929	36.9790
0.60	63.0184	59.4673	56.8628	55.85501	58.2238	52.1899
0.75	75.6253	75.56751	75.5542	73.12894	69.6130	68.2240
slope	10[illegible]205	99.8783	97.7300	95.0861	93.5423	87.0410

(Table A1 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.2523	6.088888	6.0044	5.814763	5.6625	5.3142
0.30	14.7866	14.07268	13.3875	13.00313	12.6960	11.5630
0.45	22.4986	22.21325	21.9438	21.34868	20.8516	18.1265
0.60	32.3408	30.30576	29.1889	28.28132	28.5723	25.5814
0.75	38.8583	38.60455	38.9767	37.62002	35.8472	33.5140
slope	51.7547	50.3176	49.7273	48.1324	46.9253	42.7534

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	12.1729	11.84982	11.6715	11.29958	11.0003	10.1368
0.30	29.2534	27.31395	26.0330	25.27986	24.6671	22.1540
0.45	42.1999	43.94956	42.6144	41.44941	40.4741	35.2021
0.60	59.7403	56.41921	54.0933	53.16018	55.4650	49.6744
0.75	71.5365	71.55432	71.6469	69.34348	66.1916	64.9949
slope	96.2331	94.7606	92.8713	90.3716	89.0394	82.8755

Table A2: Model SB, L = 26/33

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	5.7544	5.3222	5.0769	4.9969	4.8451	5.4839
0.30	12.5790	11.6922	11.2934	10.6563	10.7547	10.1035
0.45	20.1300	19.3568	18.6473	17.6826	16.8739	16.8045
0.60	27.9911	26.4617	26.1423	25.1752	24.4275	25.4058
0.75	37.6497	34.4006	33.1307	32.7355	31.8271	31.1677
Slope	47.4564	44.1972	42.8883	41.6649	40.4632	40.4322

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	11.2004	10.5625	10.1647	9.8528	9.8175	10.2023
0.30	25.3168	23.5407	22.7421	21.4188	20.8846	19.8712
0.45	40.6737	38.6136	37.3182	35.5406	33.9987	30.2060
0.60	51.7697	49.6376	51.5419	50.0055	48.7495	42.2816
0.75	69.4622	63.3050	61.4267	64.1872	62.6079	55.3543
Slope	89.4842	83.4618	82.5339	82.4570	80.1964	71.0861

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	5.4390	5.1015	4.9095	4.7583	4.7204	5.2999
0.30	12.2159	11.3728	10.9855	10.3562	10.1339	9.8615
0.45	19.6587	18.6765	18.0565	17.1611	16.4127	15.3153
0.60	26.3127	24.9231	24.9421	24.2065	23.6039	23.1585
0.75	35.4214	32.3349	31.1680	31.0751	30.3182	28.8382
Slope	44.9945	41.8477	40.8071	39.8977	38.8161	37.3083

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	10.5323	9.9255	9.5483	9.2676	9.2013	9.5417
0.30	23.7691	22.1363	21.3677	20.1369	19.6235	19.0098
0.45	38.2421	36.3099	35.0942	33.3893	31.9241	27.9010
0.60	48.5310	46.6343	48.4775	47.0355	45.8562	39.6962
0.75	64.8686	59.2911	57.5041	60.3787	58.8965	52.0300
Slope	83.7895	78.3176	77.4541	77.5448	75.4095	66.6908

(Table A2 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	5.2615	4.9619	4.7751	4.6512	4.7423	5.0706
0.30	11.8851	11.0605	10.6842	10.0691	9.8126	9.5522
0.45	19.0827	18.1308	17.5298	16.6756	15.9658	14.3321
0.60	24.7777	23.4598	24.2160	23.5028	22.9184	21.4100
0.75	33.3947	30.4572	29.5379	30.1724	29.4386	26.6359
Slope	42.7108	39.7092	39.1862	38.7502	37.7129	34.6656

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	9.9355	9.3626	9.0063	8.7430	8.9327	8.9268
0.30	22.3873	20.8766	20.1529	18.9787	18.5059	17.6046
0.45	35.9300	34.1193	32.9793	31.3896	30.1225	26.3067
0.60	45.5371	43.7792	45.5634	44.2110	43.1046	37.4275
0.75	60.6606	55.6492	53.9801	56.7573	55.3673	48.8685
Slope	78.5396	73.5559	72.7762	72.9090	70.9778	62.6799

Table A3: Model SBS, L = 26/33

7.336 kip, 25.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	8.1108	8.3384	8.2606	7.8740	7.6405	6.9621
0.266	19.7542	18.4102	17.3855	17.2282	16.8255	16.0593
0.399	31.4406	30.2228	29.2588	28.3767	28.2289	24.0711
0.533	45.0880	42.2973	40.8827	39.4521	38.7610	34.6337
0.666	54.0579	54.0560	54.1838	51.1478	50.1090	45.8412
Slope	80.9118	78.5517	77.1814	73.8698	72.5807	65.4083

7.336 kip, 38.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	16.9786	16.5446	16.3280	15.8129	15.3944	13.7690
0.266	40.2224	38.2602	36.4215	35.3331	34.5013	31.0352
0.399	59.2195	61.4413	59.6654	57.8830	56.6336	49.3399
0.533	84.0493	79.4828	76.2674	79.7659	78.1025	69.4785
0.666	101.3562	101.0770	101.1204	95.0263	93.2678	91.0074
Slope	152.6595	150.2878	147.3027	143.9568	141.0518	130.6443

9.782 kip, 25.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	7.9261	7.9733	7.8994	7.5436	7.3441	6.7758
0.266	19.2113	17.8771	17.3769	16.8562	16.4576	15.3433
0.399	29.9155	28.8497	27.9895	27.6963	27.1065	23.5197
0.533	42.9545	40.2698	38.9253	37.5154	37.5663	33.2124
0.666	51.5710	51.4290	51.7753	48.9491	47.7177	43.6662
Slope	77.2505	74.8928	73.8961	70.8851	69.6949	62.7000

9.782 kip, 38.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	16.1930	15.7389	15.5281	15.0381	14.6398	13.6010
0.266	38.9844	36.4100	34.6323	33.5994	32.8109	29.5538
0.399	56.2101	58.5362	56.8486	55.1535	53.9653	46.9455
0.533	79.7174	75.3398	72.2667	76.0091	74.4319	66.1717
0.666	95.8745	95.7641	95.8087	90.0270	88.3712	86.6978
Slope	144.8728	142.5926	139.7395	136.7946	134.0465	124.4877

(Table A3 con't.)

12.227 kip, 25.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	7.9290	7.7240	7.6205	7.3801	7.1849	6.7489
0.266	18.7595	17.8445	16.9997	16.4907	16.1013	14.6825
0.399	28.5007	28.1767	27.8784	27.0579	26.4824	23.0150
0.533	40.9084	38.3942	37.1040	37.2728	36.5278	32.4392
0.666	49.1857	48.9437	49.4338	46.6100	45.5077	42.5948
Slope	73.8018	71.8525	71.1154	68.7721	67.2440	61.1555

12.227 kip, 38.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.133	15.4364	15.0311	14.8137	14.3407	13.9571	12.8735
0.266	37.1109	34.6323	33.0588	32.0582	31.2815	28.1306
0.399	53.4589	55.7450	54.1422	52.5309	51.4013	44.6979
0.533	75.5702	71.4788	68.7602	72.3995	70.9050	62.9942
0.666	90.6916	90.6780	90.8724	85.6388	84.0287	82.6097
Slope	137.3274	135.2861	132.8187	130.2371	127.5945	118.5527

Table A4: Model LB, L = 40/33

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.8652	7.1372	6.9739	7.2362	7.2587	6.4618
0.30	18.3599	17.4443	16.9434	16.1384	15.3334	14.0379
0.45	29.3486	27.7562	26.8526	26.5486	25.8715	22.6636
0.60	39.5618	39.6630	37.9552	36.8727	36.1100	33.3961
0.75	50.2692	47.5761	47.9788	49.1088	47.9607	42.5336
Slope	65.7241	63.2517	62.1980	62.0840	60.5798	54.3977

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	16.2289	15.0725	14.5300	14.5515	14.5079	13.0594
0.30	37.6504	35.8033	34.7151	33.7907	32.4174	28.5887
0.45	56.7372	52.7551	51.6766	53.3343	53.1106	46.6090
0.60	73.7602	75.2404	71.9683	69.2171	67.9198	66.8391
0.75	94.2085	89.8657	91.4331	91.7504	90.2286	85.7571
Slope	124.5848	120.6344	119.2762	118.5159	116.5450	109.8431

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.7188	7.0024	6.9097	6.9715	6.9941	6.2245
0.30	17.8723	17.0109	16.5030	15.7050	15.0565	13.5676
0.45	28.2290	26.6363	25.7713	25.4917	24.8389	22.0910
0.60	38.0971	38.1098	36.4408	35.4055	34.6734	32.0794
0.75	48.2906	45.7589	46.6276	47.1912	46.1725	41.5912
Slope	63.2717	60.8687	60.1370	59.6891	58.3248	52.8371

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	15.8994	14.4942	13.9559	13.9566	13.9182	12.5287
0.30	36.1511	34.3790	33.9667	32.4383	31.0990	27.4163
0.45	54.3044	50.4684	49.5844	52.1615	51.0356	44.7343
0.60	69.4057	71.9427	68.8571	66.1871	64.9537	64.1960
0.75	90.0624	85.8354	87.4453	87.6780	86.1851	82.3540
Slope	118.6726	115.3461	114.3391	113.7523	111.5107	105.4689

(Table A4 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.5746	6.8697	6.7812	6.7807	6.7582	6.0833
0.30	17.5240	16.6800	16.1826	15.4204	15.1002	13.3222
0.45	27.1452	25.5805	24.7637	24.8811	24.3267	21.6986
0.60	36.6809	37.0750	35.0054	33.9879	33.3100	31.2241
0.75	46.4503	44.0024	44.7585	45.4035	44.4409	40.0582
Slope	60.9737	58.8222	57.8487	57.6041	56.4100	51.2740

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	15.2917	13.9333	13.4194	13.4233	13.3656	12.0375
0.30	34.7005	33.5826	32.6185	31.1294	29.8760	26.3250
0.45	51.9566	48.4259	47.5914	50.1066	49.0272	42.9198
0.60	66.3015	68.8546	65.8545	63.4690	62.2544	61.6865
0.75	86.0716	81.9556	83.5989	83.9305	82.5143	79.1394
Slope	113.4698	110.4936	109.4355	109.0340	106.8834	101.3200

Table A5: Model LBS, L = 40/33

26.713 kip, 16.500 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	5.2755	5.4179	5.3705	5.1182	4.9654	4.5239
0.330	12.8327	11.9732	11.5374	11.1952	10.9438	10.4339
0.495	20.4460	19.6440	19.0151	18.4534	18.3300	15.6391
0.661	29.3525	27.5156	26.5580	25.6003	25.1953	22.5298
0.826	35.1074	35.1294	35.2721	33.2861	32.5593	29.7700
Slope	42.4081	41.1729	40.5210	38.7174	38.0260	34.2675

26.713 kip, 24.750 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	11.0440	10.7580	10.6152	10.2785	10.0047	8.9480
0.330	26.1294	24.8822	23.6604	22.9614	22.4416	20.1642
0.495	38.5108	39.9348	38.7760	37.6406	36.7748	32.0579
0.661	54.7139	51.7047	49.5454	48.4202	50.7676	45.1958
0.826	65.8256	65.6865	65.8254	61.8407	60.6034	59.1028
Slope	80.0119	78.7736	77.2320	73.9772	73.9001	68.4459

35.618 kip, 16.500 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	5.2768	5.1848	5.1358	4.9032	4.7726	4.4116
0.330	12.4797	11.6257	11.2881	10.9536	10.7045	9.9690
0.495	19.4549	18.7506	18.1895	18.0100	17.6009	15.2810
0.661	27.9645	26.1974	25.2872	24.4036	24.4196	21.6040
0.826	33.4933	33.4229	33.7052	31.8562	31.0065	28.3584
Slope	40.5034	39.2558	38.7454	37.1798	36.5150	32.8497

35.618 kip, 24.750 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	10.5333	10.2342	10.0955	9.7752	9.5146	8.8397
0.330	25.3261	23.6796	22.4987	21.8354	21.3427	19.2022
0.495	36.5541	38.0477	36.9464	35.8667	35.0431	30.5030
0.661	51.8955	49.0111	46.9469	46.0531	48.3830	43.0459
0.826	62.2675	62.2357	62.3686	58.5890	57.4235	56.3050
Slope	75.9329	74.7424	73.2678	70.2559	70.2320	65.2219

(Table A5 con't.)

44.522 kip, 16.500 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	5.1574	5.0223	4.9541	4.7970	4.6692	4.3856
0.330	12.1863	11.6046	11.0431	10.7162	10.4728	9.5396
0.495	18.5348	18.3134	18.1175	17.5950	17.1958	14.9532
0.661	26.6330	24.9773	24.1040	23.3188	23.7429	21.1012
0.826	31.9446	31.8083	32.1814	30.3344	29.5704	27.6613
Slope	38.6829	37.6626	37.2876	35.6633	35.2302	32.0390

44.522 kip, 24.750 Hz

SF	1068.54#	1602.80#	2065.84#	2582.30#	3098.76#	6455.74#
0.165	10.0413	9.7741	9.6310	9.3218	9.0708	8.3669
0.330	24.1092	22.5238	21.4765	20.8338	20.3479	18.2777
0.495	34.7654	36.2339	35.1879	34.1616	33.3785	29.0426
0.661	49.1965	46.4991	44.6695	43.8317	46.0910	40.9795
0.826	58.9018	58.9312	59.1561	55.7329	54.6014	53.6507
Slope	71.9790	70.9133	69.6398	66.8721	66.8517	62.1131

APPENDIX B - Summary of data for Model F

Table B1: Model F

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.8954	7.9567	7.6863	7.2106	7.1291	6.1514
0.30	18.1235	16.7513	15.8125	15.9345	15.6288	15.1816
0.45	28.8230	27.5148	26.9363	26.0286	26.5787	22.6295
0.60	39.3150	39.1351	37.6583	36.6869	35.8768	32.4759
0.75	48.0183	48.8537	49.2544	47.8262	46.6812	43.0371
Slope	63.9955	63.6136	62.6697	60.9750	60.0043	54.4839

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	15.6560	15.6955	15.1687	14.6979	14.3238	12.7859
0.30	36.1090	33.6002	33.2585	32.3418	31.6423	29.1711
0.45	53.8523	51.8264	52.6217	51.1427	52.0236	45.4052
0.60	77.1726	73.1771	70.3431	68.7074	67.3839	63.9906
0.75	93.3833	95.0017	92.5846	89.6961	87.0836	83.9613
Slope	124.2470	121.9505	119.2541	115.8933	113.7737	107.0440

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.6006	7.6837	7.4226	6.9690	6.8157	6.0474
0.30	17.4612	16.1299	15.2183	15.3387	15.0171	14.7941
0.45	27.7596	26.5021	25.9723	25.0919	25.4454	21.7966
0.60	37.7353	37.7127	36.2894	35.3497	34.4744	31.3103
0.75	46.2129	46.9098	47.4665	46.0933	44.8990	41.5190
Slope	61.5524	61.1939	60.3959	58.7621	57.6459	52.5893

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	15.0276	15.0960	14.5925	14.1371	13.7703	12.2963
0.30	34.7248	32.2892	31.9543	31.0854	30.4066	27.9583
0.45	51.7723	49.8015	50.6323	49.1839	50.0847	43.6672
0.60	73.9152	69.7237	66.7460	65.1656	63.6517	61.5868
0.75	90.3126	91.0161	88.6260	85.0584	81.7697	80.6869
Slope	119.6385	116.7338	114.0015	110.2804	107.6719	102.9086

(Table B1 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	7.3201	7.4187	7.1653	6.8329	6.6597	5.9444
0.30	16.8233	15.6359	15.4732	15.0464	14.7190	14.3061
0.45	26.7302	25.5608	25.0337	24.1837	24.5306	21.1075
0.60	36.2431	36.3354	34.9587	34.0550	33.2822	30.1752
0.75	44.4674	45.0318	45.7280	44.4152	43.2724	40.0407
Slope	59.2081	58.8937	58.3864	56.6997	55.6582	50.7616

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	14.4484	14.5159	14.0275	13.5944	13.2726	11.8306
0.30	33.3854	31.0204	30.7098	29.8691	29.2302	26.9124
0.45	49.7593	47.8747	48.6845	47.3078	48.2118	41.9781
0.60	70.8399	66.8726	64.1160	62.6621	60.7416	59.2508
0.75	86.5070	87.1686	84.9972	81.5211	78.6681	77.6632
Slope	114.7141	111.9411	109.4486	105.8800	103.3546	99.0193

Table B2: Model SF, L = 30.83/40.3

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.7036	6.1083	5.8491	5.6251	5.4168	5.4677
0.30	14.1910	13.3732	13.8194	13.3730	13.3716	11.5850
0.45	23.3392	22.7161	21.4325	20.2020	19.7089	19.0273
0.60	32.7401	30.6351	30.6198	29.6457	28.9377	24.4131
0.75	40.8710	40.3625	38.7749	37.2891	37.3676	31.8318
Slope	53.3841	51.5583	50.1987	48.2431	47.7424	41.5189

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	13.0259	12.2935	11.8366	11.4366	11.1144	10.3911
0.30	28.3981	26.9607	26.3061	25.4951	25.0667	22.4176
0.45	42.9678	44.3760	42.0687	40.0356	39.0370	35.7176
0.60	60.4568	56.3495	59.2834	57.5735	56.1786	48.6454
0.75	77.0273	74.0650	70.8951	68.5409	72.1590	62.5531
Slope	100.0835	96.3716	94.8199	91.5796	92.5901	81.1789

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.4078	5.8592	5.6424	5.4517	5.2982	5.4087
0.30	13.5565	12.8672	13.1863	12.7601	12.8070	11.0815
0.45	22.2844	21.6649	20.4299	19.2411	18.7725	18.2693
0.60	31.1955	29.1849	29.2146	28.2831	27.6094	23.2502
0.75	38.7931	38.4709	36.9704	35.5355	35.6591	30.3330
Slope	50.8026	49.1736	47.8806	46.0006	45.5712	39.6418

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	12.3834	11.6874	11.2515	10.8712	10.5648	9.7370
0.30	26.9640	25.6038	25.0068	24.2375	23.8617	21.3389
0.45	40.7873	42.2337	40.0045	38.0391	37.0928	34.0171
0.60	57.2034	53.4301	56.4321	54.7894	53.4416	46.2398
0.75	72.9456	70.0998	66.4336	64.6349	68.6343	59.4643
Slope	94.8140	91.3715	89.5969	86.7631	88.0611	77.1814

(Table B2 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	6.1230	5.7308	5.5185	5.3320	5.1818	5.2559
0.30	13.2529	12.5800	12.5779	12.1711	12.2660	10.6092
0.45	21.2747	20.7499	19.6099	18.6721	18.2053	17.4971
0.60	29.7142	27.8322	27.8642	26.9737	26.3328	22.6783
0.75	36.8164	36.6568	35.2402	33.9111	34.0174	29.1596
Slope	48.4111	47.0004	45.7166	44.0171	43.6058	38.2396

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	11.7821	11.1105	10.7375	10.3703	10.0751	9.3367
0.30	25.6789	24.3529	23.7615	23.0322	22.7069	20.3048
0.45	38.7383	40.1803	38.0261	36.1258	35.2294	32.3871
0.60	54.2978	50.7543	53.6991	52.1402	50.8604	43.9341
0.75	69.2689	66.4579	63.1166	61.4134	65.3230	56.5039
Slope	90.0473	86.7471	85.1779	82.4775	83.7859	73.3774

Table B3: Model SFS, L = 30.83/40.3

6.715 kip, 26.144 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	10.3222	10.3997	10.0491	9.4264	9.2149	8.0442
0.262	23.6627	21.8896	21.4968	20.8213	20.3768	19.9950
0.394	37.7207	35.9516	35.1976	34.0330	34.4967	29.5653
0.525	51.3854	51.1835	49.2466	47.9765	47.0495	42.3936
0.656	62.6362	63.7853	64.3790	62.5644	60.8813	56.2653
Slope	95.5560	95.0241	93.8909	91.1718	89.5320	81.4344

6.715 kip, 39.216 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	20.4780	20.5245	19.8405	19.2249	18.7303	16.7214
0.262	47.1614	43.9264	43.5033	42.2808	41.3490	38.1384
0.394	70.4125	67.7339	68.7857	66.9084	68.0417	59.3429
0.525	100.8662	95.6752	91.9558	89.8175	88.6724	83.5612
0.656	121.7879	124.0390	121.0514	117.2878	113.7914	109.8237
Slope	185.4826	182.1611	178.2545	173.2567	170.3437	159.9764

8.954 kip, 26.144 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	9.9367	10.0427	9.7042	9.1726	8.9069	7.9082
0.262	22.7980	21.0776	20.6947	20.0429	19.6156	19.3409
0.394	36.3306	34.6167	33.9377	32.8084	33.2640	28.4770
0.525	49.3212	49.3236	47.4564	46.2280	45.3635	40.8715
0.656	60.2813	61.2480	62.0425	60.2992	58.6857	54.2806
Slope	91.9091	91.4055	90.4857	87.8731	86.3100	78.5630

8.954 kip, 39.216 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	19.6568	19.7413	19.0840	18.4919	18.0143	16.0817
0.262	45.3552	42.2141	41.8117	40.6399	39.7489	36.5678
0.394	67.6950	65.0702	66.1723	64.3206	65.4800	57.0734
0.525	96.6126	91.2063	87.2316	85.2227	83.1333	80.4251
0.656	117.7894	118.8399	115.8756	111.2264	106.9383	105.4703
Slope	178.6100	174.3909	170.3892	164.8775	160.9170	153.7541

(Table B3 con't.)

11.192 kip, 26.144 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	9.5654	9.7615	9.3692	8.9370	8.7080	7.7743
0.262	21.9806	20.4401	20.2398	19.6692	19.2332	18.7055
0.394	34.9853	33.3847	32.7165	31.6213	32.0686	27.5871
0.525	47.3478	47.5230	45.7235	44.5355	43.7952	39.5882
0.656	58.0059	58.7972	59.7802	58.1062	56.5604	52.3562
Slope	88.4012	87.9814	87.2773	84.7855	83.3375	75.9534

11.192 kip, 39.216 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.131	18.8930	18.9836	18.3522	17.7829	17.3496	15.4687
0.262	43.6080	40.5576	40.1750	39.0522	38.2006	35.2067
0.394	65.0660	62.5555	63.6493	61.8704	63.0345	54.8774
0.525	92.5754	87.4555	83.8766	81.9524	79.9470	77.3902
0.656	112.8317	113.8229	111.1283	106.6070	102.8457	101.5663
Slope	171.2538	167.2259	163.6361	158.3070	154.7770	147.9915

Table B4: Model LF, L = 52.4/40.3

15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	9.2239	9.0961	9.4559	9.1865	8.9177	7.7425
0.30	21.4240	21.0707	20.5218	19.4841	18.4337	18.6598
0.45	33.5920	33.7701	32.6639	31.6999	31.5862	28.5981
0.60	45.0052	45.9197	44.0506	45.1511	44.4058	40.2415
0.75	57.9547	56.7603	54.1819	56.6683	55.2143	52.3225
Slope	75.4718	75.1549	72.1943	73.6001	72.0289	67.0830

15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	19.1580	18.4529	18.8436	18.3339	17.9139	16.1574
0.30	41.5698	42.6825	41.4495	39.4674	37.7665	36.2647
0.45	68.0568	64.3919	61.8317	60.5824	59.7873	58.0517
0.60	87.5063	86.2677	85.4399	85.0238	83.6187	78.8558
0.75	114.0904	109.8272	107.1774	106.1931	106.6532	98.0492
Slope	148.7206	144.3879	141.1981	139.4032	138.2484	129.5165

20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	9.0949	8.8322	9.2026	8.9401	8.6773	7.6410
0.30	20.7934	20.4675	19.9360	18.9217	17.8987	18.1445
0.45	32.5649	32.7974	31.7235	30.8064	30.7248	27.7834
0.60	43.7032	44.6428	42.7540	43.8698	43.1467	39.1307
0.75	56.2038	55.1300	52.5864	55.0861	53.5176	50.8470
Slope	73.2373	73.0159	70.0840	71.5287	69.9180	65.2167

20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	18.6073	17.8757	18.2821	17.7881	17.3811	15.6758
0.30	40.3066	41.4119	40.2172	38.2717	36.6008	35.1924
0.45	65.8022	62.2919	59.9808	58.7740	58.0124	56.3481
0.60	84.5752	82.3509	82.6925	82.2772	80.9244	76.4148
0.75	110.1058	106.0362	103.6617	102.5503	103.1458	94.2187
Slope	143.6918	139.0496	136.6954	134.8502	133.8238	125.0736

(Table B4 con't.)

25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	8.9674	8.6336	8.9548	8.6992	8.4421	7.5405
0.30	20.2044	19.9502	19.3947	18.4335	17.6556	17.6412
0.45	32.0254	31.8485	30.8062	29.9518	29.8833	27.0934
0.60	42.4342	43.3981	41.4893	42.6195	41.9190	38.0458
0.75	54.4996	53.5409	51.0314	53.5435	51.8650	49.4075
Slope	71.2348	70.9547	68.0336	69.5293	67.9277	63.4332

25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#
0.15	18.0694	17.3472	17.7345	17.2560	16.8616	15.2157
0.30	39.0750	40.1729	39.0156	37.1058	35.4643	34.1465
0.45	63.7658	60.3919	58.1760	57.0104	56.2814	54.6865
0.60	81.7877	80.3558	80.2083	79.6055	78.3021	74.1707
0.75	106.2445	102.5669	101.2063	100.3405	100.0079	91.3780
Slope	138.8957	134.9244	132.9889	131.2271	129.6827	121.3504

Table B5: Model LFS, L = 52.4/40.3

32.955 kip, 15.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	6.0752	6.1221	5.9149	5.5470	5.4214	4.7350
0.342	13.9513	12.8890	12.6474	12.2597	12.0058	11.7678
0.513	22.1795	21.1641	20.7411	20.0300	20.3099	17.4200
0.684	30.2228	30.1049	29.0076	28.2241	27.7363	24.9635
0.855	36.8944	37.5805	37.9187	36.8121	35.8450	33.1108
Slope	43.1493	42.9191	42.4239	41.1542	40.4546	36.7769

32.955 kip, 23.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	12.0537	12.0836	11.6794	11.3142	11.0211	9.8426
0.342	27.8062	25.8666	25.5971	24.8969	24.3632	22.4446
0.513	41.4055	39.8758	40.5371	39.3825	40.0627	34.9664
0.684	59.3278	56.2689	54.1596	52.8338	52.2686	49.2082
0.855	71.7365	73.0804	71.3011	69.0047	66.9934	64.6347
Slope	83.7596	82.2760	80.5439	78.2035	76.9664	72.2526

43.490 kip, 15.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	5.8484	5.9120	5.7120	5.3977	5.2402	4.6550
0.342	13.4415	12.4109	12.1755	11.8014	11.5574	11.3829
0.513	21.3622	20.3784	19.9987	19.3093	19.5842	16.7789
0.684	29.0088	29.0111	27.9532	27.1956	26.7412	24.0673
0.855	35.5080	36.0857	36.5426	35.4795	34.5524	31.9430
Slope	41.5029	41.2848	40.8854	39.6653	38.9983	35.4802

43.490 kip, 23.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	11.5704	11.6225	11.2341	10.8829	10.5998	9.4661
0.342	26.7415	24.8584	24.6019	23.9308	23.4205	21.5235
0.513	39.8077	38.3068	38.9954	37.8577	38.5527	33.6294
0.684	56.8262	53.6490	51.3839	50.1382	49.0081	47.3615
0.855	69.3811	70.0176	68.2512	65.4472	62.9647	62.0699
Slope	80.6564	78.7700	76.9917	74.4286	72.7116	69.4417

(Table B5 con't.)

54.925 kip, 15.385 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	5.6294	5.7464	5.5147	5.2597	5.1239	4.5762
0.342	12.9604	12.0365	11.9091	11.5823	11.3324	11.0087
0.513	20.5708	19.6519	19.2788	18.6104	18.8802	16.2552
0.684	27.8460	27.9516	26.9321	26.1995	25.8164	23.3113
0.855	34.1672	34.6413	35.2097	34.1886	33.3006	30.8101
Slope	39.9178	39.7376	39.4354	38.2714	37.6550	34.3016

54.925 kip, 23.077 Hz

SF	293.45#	440.17#	567.33#	709.16#	851.00#	1772.91#
0.171	11.1202	11.1763	10.8032	10.4655	10.2081	9.1047
0.342	25.7110	23.8826	23.6386	22.9956	22.5080	20.7221
0.513	38.2612	36.8259	37.5081	36.4151	37.1124	32.3350
0.684	54.4498	51.4408	49.4070	48.2135	47.1291	45.5738
0.855	66.4599	67.0607	65.4541	62.7282	60.5542	59.7716
Slope	77.3329	75.5321	73.9393	71.4616	69.9363	66.8381